An Econometric Identification of Abnormally Low Bids in Procurement Market: Discriminant Analysis

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October 2018

Abstract

In the public construction procurement market, 'abnormally low bids (ALB)' are prevalent and they cause many social and economic problems. Also, when the procurement bids are colluded, ALB make the competitive price systematically underestimated. As many countries regulate ALB, their criteria to identify ALB are not homogenous. Most of the criteria are based on construction cost, which is usually inaccurate, vulnerable to accounting manipulation, and limited to the supply side information of the market. We propose an econometric identification process of ALB using a discriminant analysis. It is based on a switching regression with incomplete separation information and easily estimable by MLE. Through a Monte Carlo simulation, we show that our new method works well. We apply our method to Korean public construction bidding data from 2007 to 2016. The estimation results identify the determinants of the bid prices, along with the determinants of ALB, and presents a more accurate assessment of the collusion damage.

JEL Classification: H57, L40, L70

Keywords: abnormally low bids, discriminant analysis, public procurement market

Acknowledgement: We are grateful to Yun J. Choi, Jiyoung Lee, and the participants of Yonsei School of Economics Wednesday Seminar for their helpful advice and comments. We also thank Construction Association of Korea (CAK) for the valuable data. This study was partially funded by the National Research Foundation of Korea (Grant Number NRF-2016S1A3A2923769). The authors declare that they have no conflict of interest.

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1. Introduction

The public construction bidding market is unique in two aspects. First, collusions are prevalent in the market. Because the market is usually a monopsony, the competition between suppliers becomes fierce and the winning bid price converges to the marginal cost of the most efficient supplier. Such a severely competitive environment can provide a strong incentive to form a cartel, and thus collusions are often observed. Second, 'abnormally low bids (ALB)' are likely to occur in the market. This is because public construction contracts are typically long-term and huge in scope.

The legal definition of ALB is not uniform and somewhat ambiguous. In economic terms, ALB could be defined as 'a significantly lower bid than the bidder's marginal cost.' There are many reasons for ALB: predatory pricing, underestimation of the cost or risk of construction, the intention to change its plan after winning the bid, and etc. We will elaborate these in the next section.

ALB cause many problems. First of all, the competitive price which maximizes the social welfare cannot be achieved when ALB exist in the market. That is the main reason why a lot of countries ban ALB. Another problem of ALB is that they make it difficult to precisely measure the damage of collusion. In order to assess the damage, the competitive price needs to be set as the standard to be compared to a suspected collusion price. When ALB exist in the bidding, however, they erroneously lower the competitive price so that the collusion effect is overestimated.

Nevertheless, it is not an easy task to identify ALB from the bids in public construction procurements. Usually, a 'normal' price is calculated from the construction costs, and it is compared to the actual bids to see if any bid is too much lower than the 'normal' price. This cost-based identification has a few problems. First, the actual construction costs are rarely observed with accuracy. Second, the cost data are often subject to accounting manipulation. Third, the cost information only reflects the supply side of the market equilibrium, ignoring the demand side information at all.

We propose a discriminant analysis to econometrically detect ALB using statistical data on the market, without utilizing specific cost information. Discriminant analysis is first formulated by Fisher (1936), and developed by Goldfeld and Quandt (1972), Kiefer (1980), Quandt and Ramsey (1978) and Schmidt (1982). In the field of law economics, Porter (1983), Lee and Porter (1984), and Ellison (1994) use this method to detect the collusion periods from weekly time series data on the Joint Executive Committee (JEC) railroad cartel from 1880 to

1886. To be exact, the methodology they use is "Switching Regression Model with Imperfect Sample Separation Information." We apply a general switching regression model similar to Ellison (1994) and repeat it to elicit the most realistic sample separation information between regression regimes.

We present a Monte Carlo simulation to investigate the finite sample properties of our method. In addition, we apply our method to Korean public construction bidding data from 2007 to 2016, and empirically identify ALB.

2. Abnormally Low Bids

The definition of ALB varies. Albano (2017) surveys the various standards of ALB in the international law. The 'UNCITRAL Model Law on Public Procurement 2011' by the United Nations states: "The procuring entity may reject a submission if the procuring entity has determined that the price, in combination with other constituent elements of the submission, is abnormally low in relation to the subject matter of the procurement and raises concerns with the procuring entity as to the ability of the supplier or contractor that presented that submission to perform the procurement contract." The World Trade Organization states: "Only tenders that conform to the essential requirements of the tender notice or documentation and are from a supplier which complies with the conditions for participation can be considered for award. Entities have the obligation to award contracts to the tenderer who has been determined to be fully capable of undertaking the contract and whose tender is either the lowest tender or the tender which is determined to be the most advantageous in terms of the specific evaluation criteria set forth in the notices or tender documentation. An entity that has received a tender abnormally lower than other tenders may enquire with the tenderer to ensure that it can comply with the conditions of participation and be capable of fulfilling the terms of the contract." (Article XIII: 4).

According to the Federal Acquisition Regulation of the U.S. government, ALB can be referred to as unfair and unreasonable bids compared to the preliminary estimate of a client.[3] The European Union explicitly obliges its member states to explain the price or costs contained

[3]Carpineti et al. (2006). Although the U.S. Federal Acquisition Regulation does not define ALB explicitly, it emphasizes that all prices during the procurement procedure should be fair and reasonable. The footnote 27 in Carpineti et al. (2006) explains shat 'fair and reasonable prices' are.

in a tender in situations where tenders "appear to be abnormally low in relation to the works, goods or services". The European Union provides the following guidelines as to the elements of a tender price may be subjected to further scrutiny:[4]

- in the light of client's preliminary estimate & of all the tenders submitted, it seems to be abnormally low by not providing a margin for a normal level of profit
- In relation to which the tenderer cannot explain his price on the basis of the economy of the construction method, or the technical solution chosen, or the exceptionally favorable conditions available to the tenderer, or the originality of the work proposed.

In sum, ALB can be legally defined as the bid whose price is unreasonably low so that the bidder cannot profit from the contract. In economic terms, ALB could be defined as 'a significantly lower bid than the bidder's marginal cost.'

The reasons why ALB occurs in construction procurements are manifold. OECD (2016) and Ibrahimi (2017) list the following reasons. First, the bidder misunderstands the conditions or details of construction contract. Second, the bidder underestimates the risk of the contract and submits a very low price. Third, the bidder pursues illegal profits by not complying with the essential laws involved with the construction. Fourth, a government subsidy can make a bid look like an ALB as the bidder offers a lower price than the other bidders without subsidy.

Sometimes, bidders use ALB as a strategy. First, a company would take the risk of a significant loss if it is desperate in winning the contract due to cash flow problem. As public constructions contracts are typically long-term, some companies may take the short-term loss to stabilize the income stream. Gunduz and Karacan (2008) survey on the causes of ALB in Turkish government procurements. They find that the hope of staying in the business is the most important reason. Calveras et al. (2004) also argue that a firm in financial trouble bids more aggressively with a lower price in order to survive in the market.

Second, firms utilize ALB as a predatory pricing strategy. In other words, they submit an abnormally low bid to drive competitors out of the market. Alexandersson and Hulten (2016) find ALB as predatory pricing by analyzing Swedish train service data in 2002. Bedford (2009) also argues that firms' predatory intentions cause ALB and that a prequalification procedure can reduce predatory ALB. Third, when the bidder considers re-negotiations after winning the

[4] See Harrower (1999) for a detailed explanation.

contract, ALB may occur. Calveras et al. (2004) claim that one of the main reasons for ALB is the expectation for re-negotiation at the time the client cannot change the contractor.

ALB makes a number of problems. First, the contractor with ALB faces many risks: a default risk, a risk of paying additional costs during implementing the contract, a risk of not abiding by the laws or the contract terms.[5] Second, quality deterioration is also possible due to ALB. If the quality of the construction becomes worse, the consumers' satisfaction and social welfare will be lowered. Third, ALB can remove the competitors out of the market by predatory pricing. Such reduction in competition would be harmful to consumers.

In addition to these negative effects, ALB makes it difficult to assess the damage of collusion in the market. The damage of collusion is usually measured by the difference between the competitive price and the actual price. As the price of ALB is most likely a low-end outlier, the existence of ALB in the market tends to bias the estimated competitive price downward. Accordingly, the collusion damage would most likely to be overestimated.

For these reasons, many countries regulate ALB. Of course, the major problem in the regulation is the difficulty in identifying ALB. All the current regulations are based on the observed bid price. The criterion to determine a bid 'too low,' however, varies country by country. For example, Belgium determines a bid to be an ALB if the bid is lower than 85% of the mean bid. Portugal compares the bids to the estimated base price. If the bid is lower than 60% of the base price, the bid is determined as an ALB.

In general, the criteria for ALB could be categorized by 'absolute criterion' and 'relative criterion.' An absolute criterion evaluates the deviation of a bid price from the client's preliminary estimate of the price. If the bid is lower than the allowed deviation, it is determined 'abnormally low.' Though the absolute criterion can be applied regardless of the number of bidders, it depends on the accuracy of the estimated price. A relative criterion uses the deviation from the mean of the other bid prices. To avoid a distortion, most countries preset the minimum number of bidders, or exclude the maximum and minimum bids. While the relative criterion does not need any pre-determined price estimate, it is difficult to apply when the number of bidders is small. Some countries combine both the absolute and relative criteria. Table 1 summarizes the criteria of ALB for selected countries.

[5] OECD(2016), Ibrahimi(2017)

<Table 1 > ALB Criteria in Selected Countries

Country	Type of Criteria	Applied Cases	Criteria	Notes
World Bank	Absolute	bidders<5	20% or more below the Borrower's cost estimate	World Bank (2016)
	Relative	bidders≥5	More than one standard deviation below the average of the substantially responsive bids received	
Belgium	Relative	bidders≥4	15% lower than the mean of the bids if at least 4 bids are submitted (the mean refers to the mean of all the bids apart from the highest and the lowest if the bids are equal or more than 7)	
Bulgaria	Relative	-	20% lower than the mean of the other bids (30% until 2012)	Public Procurement Act, Article 70.
Hungary	Absolute	-	20% lower than the available funds (more flexibility introduced after 2015)	Act CXLIII of 2015 on Public Procurement, Article 69.
Italy		-	Points scored in price and quality are both more than 4/5 of the corresponding maximum points (which implies low price for high quality)	OECD (2016)
Portugal	Absolute	-	40% lower than the base price in the specifications	
Romania	Absolute	bidders<5	Less than 85% of the estimated value of the contract	Until 2016
	Relative	bidders≥5	Less than 85% of the arithmetic average of the price of the submitted tenders, without taking into account the lowest and highest prices proposed	
Slovenia	Relative	bidders≥4	50% lower than the mean of the bids and 20% lower than the 2nd lowest bid if at least 4 bids are submitted	The Public Procurement Act (ZJN-3), Official Gazette no. 91/15, Article 86.
Spain	Absolute	1 bidder	25% lower than the base price of the contract	
	Relative	2 bidders	20% lower than the second bid if 2	

			bids received	
	Relative	3 bidders	10% lower than the mean of all the bids, but if the highest bid is 10% higher than the mean of all bids it should be excluded from the calculation of the mean	
	Relative	bidders≥4	10% lower than the mean of all the bids, but all bids that are higher than 10% from the mean of all bids should be excluded from the calculation of the mean and if the remaining bids are less than 4 then one of the above three rules should be applied.	
Poland	Absolute Relative	-	Lower by at least 30% from the contract value or the arithmetic mean of the prices of all tenders submitted	ACT of 29 January 2004 PUBLIC PROCUREME NT LAW
Slovakia	Relative	bidders≥3	At least 15% lower than the average price of all other offers or at least 10% lower than the second lowest offer.	
Turkey		-	Uses a preset formula based on the arithmetic mean of the bids (by excluding bids which are over %120 of conceptual cost and below %40 of conceptual cost)	Public Procurement Law, Article 38 / Karacan(2008)
Brazil	Absolute Relative	-	70% lower than the lowest of the following values: (1) the arithmetic average between tendering prices that are superior than 50% of the estimated price set by the Administration; (2) the estimated price set by the Administration.	Carpineti et al. (2006)
U.S.A.	Absolute	-	Lower than 75% or higher than 150% of the government estimate (Wisconsin) Lower or higher by more than 15% of the government estimate (New York)	Choi (2010), Downing (2004)
Japan	Absolute	-	Lower than 70%~85% of the estimated price (different by provinces)	Choi (2010, 2011)

Conti and Naldi (2008) and Ballesteros-Pérez et al (2015), among others, mathematically simulate the accuracy and efficiency of the above screening strategies for ALB.

They show that many factors such as the number of bids and the dispersion of bids affect the performance of the criteria, and that the criteria could often be significantly misleading. Fuentes-Bargues et al (2016) argues that the above ALB criteria using the price of bids would be risky and other factors, such as labor involved in the contract, economic improvements and the guarantee period need to be considered along with the price.

3. Discriminant Analysis

This paper proposes a discriminant analysis for econometrical identification of ALB from procurement data. Discriminant analysis is a statistical method separating a distribution from a mixture of distributions. Fisher (1936) first shapes up the method, and a number of procedures have been developed by Goldfeld and Quandt (1972), Kiefer (1980), Quandt and Ramsey (1978) and Schmidt (1982), among others. Our method utilizes a switching regression model to separate ALB from the mixture distribution of normal bids and ALB. Porter (1983), Lee and Porter (1984), and Ellison (1994) use this framework, so-called 'Switching Regression Model with Imperfect Sample Separation Information,' to detect the collusion periods from weekly time series data on the Joint Executive Committee (JEC) railroad cartel from 1880 to 1886. We apply a general switching regression model in line with Ellison (1994) and repeat it to elicit the most realistic sample separation information from the regression regimes.

We consider the following price equation in reduced form.

$$P_t = \beta' X_t + \delta I_t + u_t \tag{1}$$

where P_t is the price (or the price ratio to the preliminary estimate) of a winning bid, X_t is a set of variables explaining the winning price of bid *t*, for example, a dummy variable for collusion (1 if there is a collusive behavior in bid *t*, 0 otherwise), the number of bidders, the business cycle index, bidding types or period effects, etc. I_t is a dummy variable for ALB; 1 if it is ALB, 0 otherwise. u_t is an error term of the regression model. As the ALB dummy will lower the price, the coefficient δ is expected to have a negative value.

In reality, ALB is not fully observed, although there exist various criteria for ALB as in Table 1. Thus, we need another dummy variable for ALB in addition to I_t above. We define I_t and D_t as follows.

1) If bid *t* is observed as an ALB, then $D_t = 1$; otherwise $D_t = 0$.

2) If bid *t* is actually an ALB, then $I_t = 1$; otherwise $I_t = 0$.

It is important that D_t is observed with measurement error while I_t is not observed at all. In other words, D_t is a error-ridden measure of unobservable I_t. We assume the following transition probability matrix about the relation between D_t and I_t:

	$D_t = 1$	$D_t = 0$
$I_t = 1$	p_{11}	p_{10}
$I_t = 0$	p_{01}	p_{00}

where $p_{11} \equiv \Pr(D_t = 1 \mid I_t = 1)$, $p_{01} \equiv \Pr(D_t = 1 \mid I_t = 0)$, $p_{10} = 1 - p_{11}$, and $p_{00} = 1 - p_{01}$. If $p_{11} = 1$, all the true ALB are perfectly observed as ALB. On the contrary, if $p_{11} = 0$, none of the true ALB is observed as ALB. The unobservable I_t is assumed to be have the following binomial distribution:

$$I_t = 1 \quad \text{with probability } \lambda_t \tag{2}$$

$$I_t = 0 \quad \text{with probability } 1 - \lambda_t \tag{3}$$

In (2), λ_t is the unconditional probability that bid *t* is actually an ALB. In discriminant analyses, this unconditional probability is often assumed to be a constant, as in Lee and Porter (1984). It is unrealistic, however, to assume all of the bids should have the same probability of $I_t = 1$. For example, if there are more bidders in the bidding, ALB are more likely to occur because of the competitiveness. Types of bidding or the scope of work might also change the possibility of ALB. To incorporate the possibility of heterogeneous unconditional probability of ALB, we specify λ_t as a function of multiple covariates. As λ_t is a probability function, we employ a logistic function similar to Ellison (1994).[6]

[6] As Ellison (1994) deals with time-series data, he uses a Markov structure for the logit function. We employ a contemporary logit structure, as we apply our method to cross-section data.

$$\lambda_t = \frac{e^{\alpha' Z_t}}{1+e^{\alpha' Z_t}} \tag{4}$$

where Z_t is a set of variables which affect the occurrence of ALB.

From equation (1) through (4), we can derive the likelihood function of the data generation process. The likelihood function of the regression model is as follows:

$$L = \prod [f_1(P_t)\lambda_t p_{11} + f_2(P_t)(1-\lambda_t)p_{01}]^{D_t} \cdot [f_1(P_t)\lambda_t(1-p_{11}) + f_2(P_t)(1-\lambda_t)(1-p_{01})]^{1-D_t} \tag{5}$$

where $f_1(P_t)$ is the probability density function of bid price if the bid is actually an ALB (i.e. I_t=1), while $f_2(P_t)$ is the probability density function if the bid is not an ALB (i.e. I_t=0). We assume that the bid price follows a normal distribution as follows:

$$f_1(P_t) = \frac{1}{\sqrt{2\pi}\sigma}\exp\{-\frac{1}{2\sigma^2}(P_t - \beta' X_t - \delta)^2\} \tag{6}$$

$$f_2(P_t) = \frac{1}{\sqrt{2\pi}\sigma}\exp\{-\frac{1}{2\sigma^2}(P_t - \beta' X_t)^2\} \tag{7}$$

After substituting (6) and (7) into (5), the parameters in the regression model, $\beta, \alpha, \sigma^2, p_{11}$, and p_{01} are estimated by maximizing the likelihood function (5).[7] With the estimates of the parameters, we can calculate the estimated conditional probabilities, $\Pr(I_t = 1 | P_t, D_t)$ and $\Pr(I_t = 0 | P_t, D_t)$ for each observation. By comparing the two conditional probabilities, we can decide which distribution the observation belongs to. Lee and Porter (1984) show that the simplest rule is the best: if $\Pr(I_t = 1 | P_t, D_t) > \Pr(I_t = 0 | P_t, D_t)$ then the bid is discriminated as an ALB, and if $\Pr(I_t = 1 | P_t, D_t) < \Pr(I_t = 0 | P_t, D_t)$ then the bid is discriminated as a normal bid.[8] As the sum of the two conditional probabilities must be 1, the rule can be also stated as: the bid is discriminated as an ALB if $\Pr(I_t = 1 | P_t, D_t) > 0.5$.

It is noted that the above estimation needs an initial observation of D_t. The simplest procedure would be to use one of the criteria listed in Table 1. For example, if we employ the World Bank's criterion, D_t will have a value of one when the bid is lower than 80% of the

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[7] We use GAUSS and R for the numerical maximization.

[8] Lee and Porter(1984), pp 400-401.

estimated cost for the bids with less than 5 bidders, or when the bid is lower than the average by one standard deviation for the bids with more than 4 bidders. Then with the D_t constructed, we can proceed to maximize the likelihood function of (5).

Of course, if we employ another country's criterion, we may have a different initial values for D_t. Another option is to repeat the procedure for all the criteria listed in Table 1 and compare the estimation results. Looking at Table 1, it is clear that most criteria are defined by 'price ratio,' which is defined as 'the bid price divided by the estimated price.' Though the criteria vary, the range of the criteria is roughly between 60% and 85%. Based on this finding, the best procedure would be to try as many price ratios as possible from the applicable range for the initial values of D_t, and to compare the results.

For such a strategy, we need a standard for choosing the best D_t out of the various sets of D_t. The principle of LR (likelihood ratio) test is one possible candidate for the standard. If we use the D_t giving the highest likelihood value, it would certainly be the most realistic choice based on the given data. It should be noted, however, that D_t only provides the starting point of the discriminant analysis, and that the final estimation results may not depend on the choice of the initial D_t.

4. Monte Carlo Simulation

To verify the finite sample performance of the above discriminant analysis, we carry out a Monte Carlo simulation. The true DGP (data generation process) of the simulation model is assumed as follows:

$$P_t = \beta' X_t + \delta I_t + u_t \tag{8}$$

$$\lambda_t = \frac{e^{\alpha' Z_t}}{1 + e^{\alpha' Z_t}} \tag{9}$$

where P_t stands for the bid price. X_t consists of five hypothetical variables including a constant, CBSI (construction business survey index), the estimated construction cost, a dummy variable for collusion in bidding, and the number of bidders. CBSI is constructed from a uniform distribution ranged 70 to 100. The estimated cost is also randomly picked from a uniform distribution ranged 350 to 500. The collusion dummy takes 1 with a probability of 0.1, and takes 0 with a probability of 0.9. The number of bidders for each observation is an integer value from a discrete uniform distribution defined between 2 and 5. β is set for $(0, 0.3, 0.8, 20, -3)'$

I_t is a dummy indicator of ALB, 1 if ALB and 0 otherwise. The probability of I_t being 1 is defined as λ_t. The factors determining the switching probability of ALB regimes, Z_t, contain a constant, CBSI, the number of bidders, and the number of major competitors in the market which is an identifying variable. The number of major competitors is randomly picked from a uniform distribution whose range is (3, 5). Coefficient δ is set for -40, and α is set for $(0, -0.05, 0.8, 0.03)'$. The error term, u_t, is constructed from an *iid* normal distribution with mean of 0 and variance of 60. Simulation is performed 1,000 times for each case, and five different sample sizes (N) are employed: 50, 100, 200, 500, and 1,000.

To reflect the reality about ALB detection, we assume that there does not exist a unique measure of D_t. Instead, we create a number of alternative D_t's using a preset criterion: $D_t = 1(PR_t < \theta)$ where PR_t stands for the price ratio, which is widely used as the key variable for ALB identification in many countries. The range of θ is determined according to the distribution of the generated price ratio in each simulation. In the simulation, the lowest bound for θ is around 60% to avoid perfect multicollinearity. Then we increased θ by 1% up to around 90%. Thus, about 30 alternative θ's are tried in each simulation. We compute the maximized likelihood value for each θ, and choose the θ, say θ^*, giving the highest likelihood value. Then the D_t using θ^* is used for the maximum likelihood estimation of (8) and (9).

We examine two aspects of the discriminant analysis. First, we investigate how well the method discriminates ALB from normal bids. To do this, a measure for misclassification is employed. Second, we inspect how accurately the method estimates the parameters and their variance matrices. To show the accuracy, the size of an F-test is examined.

In discriminant analyses, the degree of misclassification is usually measured by the error rate, which is defined as follows:[9]

$$\text{Error rate} = \frac{n_{01} + n_{10}}{N}$$

[9] This error rate is called 'apparent error rate.' Some other error rates such as 'test sample error rate' or 'hold-out error rate' is also used for measuring errors in discriminant analysis.

where n_{01} is the number of observations in which actual ALB are misclassified as non-ALB, and n_{10} is the number of observations in which actual non-ALB are misclassified as ALB. Thus, it is the ratio of misclassified observations to total observations. This error rate was estimated for each step of iterations. <Table 2> shows the average error rate and its mean squared deviation (MSD) for five sample sizes.

<Table 2> Average Error Rates and MSD

Sample Size	Error Rates	
	Average	MSD
N=50	0.00480	0.009852
N=100	0.00463	0.007233
N=200	0.00442	0.004728
N=500	0.00462	0.003029
N=1000	0.00495	0.002284

From <Table 2>, the average misclassification rate is about 0.4%~0.5%. This means that only 4 or 5 observations out of 1,000 observations are misclassified about being an ALB on average. In other words, 99.5% of observations are precisely classified on average. As the MSD is ranged from 0.2% to 0.9%, these average values are pretty stable.

Second, we examine the empirical size of a test based on the discriminant analysis. We perform an F-test for the following hypothesis.

$$H_0 : \beta = \beta_0 \text{ and } \delta = \delta_0$$

β_0 and δ_0 are the true parameter values given in the simulation. We try the significance level of 1%, 5% and 10% for various sample sizes (N). The results are shown in <Table 3>.

<Table 3> Rejection Rates of $H_0 : \beta = \beta_0$ and $\delta = \delta_0$

Sample Size	Significance Level		
	1%	5%	10%
N=50	0.008	0.055	0.110

N=100	0.011	0.052	0.109
N=200	0.014	0.058	0.118
N=500	0.012	0.062	0.130
N=1000	0.008	0.056	0.130

As shown in <Table 3>, the empirical rejection rates of the F-test are quite close to the sizes of the test in all the significance levels. It is also noted that the accuracy of the test is fairly robust to the sample size. In sum, the discriminant analysis procedure proposed in this paper, including the method to choose the most realistic D_t out of many possible criteria, has been shown to work well regardless of the sample size.

5. Empirical Analysis

In this section, we apply the above discriminant analysis to actual procurement data. The empirical data are on Korean public construction bidding from 2007 to 2016. They are collected from the Public Procurement Service (PPS) of Korea and the Construction Association of Korea (CAK). Among the 864 procurement contracts in 2007-2016, 114 contracts (about 13.2%) are colluded.[10] The following variables are collected for the 864 contracts.

(1) Bid Price Ratio: the ratio of winning bid price to the contractor's preliminary estimate
(2) Collusion dummy: a collusion indicator; 1 if colluded, 0 otherwise.[11]
(3) Number of Bidders: total number of the bidders in the bid
(4) CBSI: Construction Business Survey Index by the Bank of Korea
(5) Type of Construction: dummy variables for 'architecture,' 'plant,' 'civil engineering,' 'landscaping' and 'railroad'. Plant construction is the base group for the four dummy variables.

[10] We use only the bids in which the number of bidders are less than 21. The reasons are: first, any bid with more than 20 bidders is regulated by a different Pre-Qualification standard, second, those bids with more than 20 bidders are usually very small construction project.

[11] More precisely, the dummy takes 1 if KFTC (Korean Fair Trade Commission) has decided the bid was colluded. Thus, there exists a possibility of under-detection for the collusion dummy.

(6) Type of Bidding: dummy variables for ‘turn-key,’ ‘alternative,’ ‘technical proposal’ and ‘lowest price.’ The lowest price bidding type is the base group for the three dummy variables.
(7) Year Dummies: The year of 2016 is the base group and 9 year dummies are used.
(8) ALB: an abnormally low bid indicator; 1 if it is ALB, 0 otherwise.

The descriptive statistics of the above variables are listed in <Table 4>.

<Table 4> Descriptive Statistics

	Count	Mean	Std Dev	Min	25%	Median	75%	Max
Bid Price Ratio	864	0.8878	0.1212	0.4151	0.8209	0.9449	0.9744	1
Collusion Dummy	864	0.1319	0.3386	0	0	0	0	1
Number of Bidders	864	4.2708	4.4449	2	2	2	4	20
CBSI	864	0.7040	0.1670	0.1460	0.6240	0.7160	0.8000	1.0130
Architecture	864	0.3090	0.4624	0	0	0	1	1
Railroad	864	0.0949	0.2933	0	0	0	0	1
Civil Engineering	864	0.4086	0.4919	0	0	0	1	1
Landscaping	864	0.0069	0.0831	0	0	0	0	1
Turn-key	864	0.6736	0.4692	0	0	1	1	1
Alternative	864	0.0694	0.2544	0	0	0	0	1
Technical Proposal	864	0.0995	0.2996	0	0	0	0	1
Year 2007	864	0.0174	0.1307	0	0	0	0	1
Year 2008	864	0.1157	0.3201	0	0	0	0	1
Year 2009	864	0.2813	0.4499	0	0	0	1	1
Year 2010	864	0.1250	0.3309	0	0	0	0	1
Year 2011	864	0.1331	0.3399	0	0	0	0	1
Year 2012	864	0.0822	0.2748	0	0	0	0	1
Year 2013	864	0.0926	0.2900	0	0	0	0	1
Year 2014	864	0.0498	0.2176	0	0	0	0	1
Year 2015	864	0.0613	0.2401	0	0	0	0	1

The regression model we estimate is as follows.

$$PR_t = \gamma C_t + \beta' X_t + \delta I_t + u_t \tag{10}$$

$$\lambda_t = \frac{e^{\alpha' Z_t}}{1 + e^{\alpha' Z_t}} \tag{11}$$

where PR_t is the bid price ratio, C_t is a collusion dummy. X_t is a vector of explanatory variables explaining the bid price: a constant, the number of bidders, CBSI, types of a construction, types of a bidding, and year dummies. CBSI and year dummies are included to capture the variation in the demand side of the construction business. I_t is the ALB indicator. As mentioned in section 3, I_t is unobservable. We specify the probability of I_t being 1 as in equation (11). Z_t includes a constant, the logarithm of the number of bidders, CBSI, an architecture construction type dummy and a lowest bidding type dummy.

In order to find the most realistic standard for D_t, a pre-set criterion for ALB, we begin with 57% for θ in $D_t = 1(PR_t < \theta)$ and increase θ by 1% point up to 98%.[12] For each θ, the likelihood function (5) is maximized, and likelihood value at the maximum is computed. The result is shown in <Table 5> and <Figure 1>.

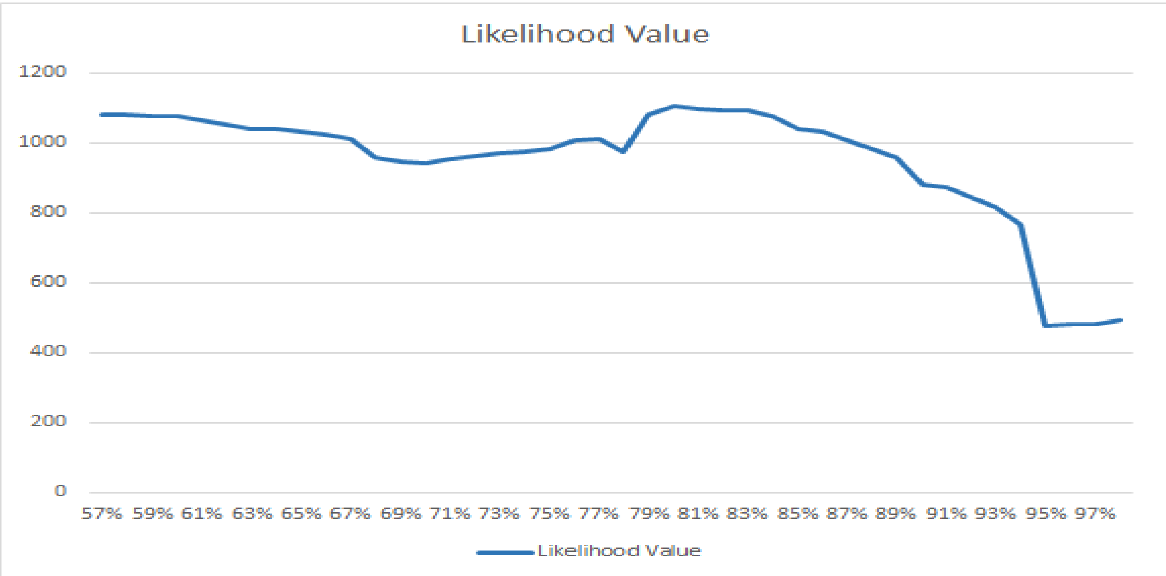
<Table 5> Maximized Likelihood Value by pre-set ALB criterion (θ)

ALB criterion (θ)	Likelihood Value	ALB criterion (θ)	Likelihood Value
57%	1080.064	78%	974.3337
58%	1080.064	79%	1080.102
59%	1079.111	80%	1105.484
60%	1075.906	81%	1097.236
61%	1065.4	82%	1092.195
62%	1053.549	83%	1092.865
63%	1040.133	84%	1076.521
64%	1039.285	85%	1042.36
65%	1033.378	86%	1030.741

[12] 57% and 98% are the lowest and highest possible θ, respectively, to ensure the existence of MLE.

66%	1025.875	87%	1005.902
67%	1010.314	88%	984.3112
68%	957.6963	89%	958.394
69%	948.0308	90%	880.5194
70%	943.5126	91%	871.7027
71%	956.394	92%	845.3267
72%	964.236	93%	815.0269
73%	972.5067	94%	766.7433
74%	974.5302	95%	477.6299
75%	984.0625	96%	478.9093
76%	1007.643	97%	481.2444
77%	1013.229	98%	490.9611
The maximized likelihood value is 1105.484 when the criterion is 80%			

<Figure 1: Likelihood Value for each ALB criterion>



As in <Table 5> above, the likelihood value is peaked at 80% with 1105.484. The maximum likelihood estimation result of equation (10) is presented in <Table 6>.

<Table 6> Maximum Likelihood Estimation Result of Equation (10)

Variable	Estimates	Standard Error	t-value
Constant	1.0275	0.0075	137.8371***
Collusion Dummy	0.0159	0.0054	2.9271***
Log(number of bidders)	-0.0030	0.0050	-0.5890
CBSI	-0.0002	0.0001	-1.4501
Architecture	0.0209	0.0053	3.9410***
Railroad	0.0267	0.0074	3.6326***
Civil Engineering	-0.0167	0.0049	-3.3815***
Landscaping	-0.0034	0.0203	-0.1653
Turn-key	-0.0333	0.0078	-4.2724***
Alternative	-0.0788	0.0088	-8.9309***
Technical Proposal	-0.0561	0.0090	-6.2148***
Year 2007	-0.0262	0.0156	-1.6798*
Year 2008	-0.0249	0.0090	-2.7635***
Year 2009	-0.0300	0.0095	-3.1557***
Year 2010	-0.0309	0.0102	-3.0386***
Year 2011	-0.0308	0.0086	-3.5794***
Year 2012	-0.0305	0.0100	-3.0655***
Year 2013	-0.0198	0.0096	-2.0599**
Year 2014	-0.0185	0.0108	-1.7209*
Year 2015	-0.0084	0.0100	-0.8423
ALB	-0.2666	0.0051	-52.5740***

(*: 10% significance level, **: 5% significance level, ***: 1% significance level)

First of all, the collusion effect estimate is 1.59% and is statistically significant at 1% level. This means that the collusive behavior makes the bid price ratio 1.59%p higher than the competitive price. This is a somewhat lower collusion effect than in other industries. Connor (2007) surveys 674 long-run cartels and finds that the median collusion overcharge is 25%. Our estimate, 1.59%, is certainly quite lower than such median. The reason might be because the client of public construction procurement is usually Korean government. Korean government tends to set the preliminary cost estimate (which is the maximum bid price) of public construction very conservatively. As the cost estimate is already very low, it is difficult for a competitive bid to be much lower than that. Thus even if the bids are colluded, there is not enough room to increase the bid highly over the competitive price. Of course, if we would not consider ALB in the estimation of the price equation, the collusion damage is estimated significantly higher than 1.59%. Though we suppress the detailed estimation result to conserve space, the collusion effect is estimated as 4.78% (three times higher than 1.59%) if the effect of ALB is not controlled. It is confirmed that ALB makes the collusion damage significantly overestimated.

Second, as the number of bidders increases by 1%, the price bid ratio falls by 0.3%. It is rational to say that the competition gets fiercer when there are more bidders. Yet, the estimate is not statistically significant. Third, the coefficient of Construction Business Survey Index (CBSI) is not statistically significant. Fourth, for construction types, compared to plant construction, the bid prices of architecture construction and railroad construction are 2.09%p and 2.67%p higher respectively and statistically significant at the 1% level. Civil construction significantly lowers the bid price ratio by 1.67%, and landscaping does not affect the price ratio.

Fifth, it is counter-intuitive that all the bidding types, turn-key, alternative, and technical proposal, produce lower price ratio than the lowest price bidding. Sixth, the year dummies compared to the base year of 2016 show the upward trend of bid price ratio, Last, when all the other variables are held constant, the ALB contracts are estimated to lower the average bid price ratio by 26.66%p.

<Table 7> presents the estimation result of equation (11), the logistic probability of ALB. It is shown that the higher the number of bidders, the higher the probability of ALB. This is plausible in the sense that a company would be inclined to ALB if the competition is severe. Second, the result shows that the probability of ALB becomes lower, if the construction industry is in prosperous business cycle. Third, among the various types of construction, architecture construction significantly lowers the probability of ALB. That a successful architecture construction requires more creativity and technology than price advantage explains the estimated coefficient. Fourth, the probability of ALB becomes higher in lowest price bidding than in any other types of bidding, which makes a perfect sense.

<Table 7> Maximum Likelihood Estimation Result of Logistic Probability (11)

Variable	Estimates	Standard Error	t-value
Constant	-3.2436	0.5808	-5.5845***
Log(number of bidders)	2.3974	0.3258	7.358***
CBSI	-1.2304	0.6221	-1.9777**
Architecture	-0.8700	0.2708	-3.213***
Lowest Price	0.9776	0.2316	4.2204***

(*: 10% significance level, **: 5% significance level, ***: 1% significance level)

As a result from the maximum likelihood estimation of the discriminant analysis model, 209 contracts out of 864 observations (about 24.2%) are classified as ALB. The detailed classification results are available from the authors upon request.

6. Conclusion

This paper proposes a discriminant analysis in order to econometrically identify abnormally low bids (ALB) which are common in public construction procurement markets. The discriminant analysis utilizes a switching regression model with imperfect sample separation information. The unconditional probability of being ALB is separately specified as a logistic function. We also suggest a procedure choosing the most informative sample separation information when there exist multiple signals on sample separation.

The Monte Carlo simulation on the finite sample performance of our procedure shows extremely low classification error rates for variable sample sizes. The empirical sizes of the F-test we perform on the parameters are also quite precise. Overall, the finite sample properties of the proposed discriminant analysis turns out to be accurate and stable.

We apply our discriminant analysis to Korean public construction procurement data between 2007 and 2016. As a result, 209 out of 864 bids are identified as ALB. After controlling the effect of ALB, we estimated the collusion effect in the market at about 1.59%p higher than the competitive price. The 1.59%p is almost one-third of 4.87%p, the collusion effect computed without considering ALB. It implies that researchers should be careful to identify ALB not to overestimate the collusion damage.

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