# Labor-Market Wedge under Engel Curve Utility: Cyclical Substitution between Necessities and Luxuries*

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**Abstract**

In booms, households substitute luxuries for necessities, e.g., food away from home for food at home. Ignoring this cyclical pattern of composition changes in the consumption basket makes the labor-market wedge—a measure of inefficiency that reflects the gap between the marginal rate of substitution and the real wage—appear to be more volatile than it actually is. Based on the household expenditure pattern across 10 consumption categories in the Consumer Expenditure Survey, we show that taking into account these composition changes can explain 6-15% of the cyclicality in the measured labor-market wedge.

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# 1 Introduction

One of the leading research questions in macroeconomics is the identification of the sources of economic fluctuations. Economists often identify these sources through accounting procedures that are based on the "wedge" between the equilibrium conditions of the model economy and the data. For example, the equilibrium models of the business cycle impose tight restrictions on the co-movement of consumption, hours, and real wages.[1] For an optimal allocation of consumption and hours worked, the marginal rate of substitution ($MRS$) between leisure and consumption has to equal the real wage. Conditional on consumption, hours worked should increases with the real wage. For reasonable parameterizations of the stand-in household's preferences, this prediction is inconsistent with observed movements in aggregate consumption, hours worked, and real wages over the business cycle. On the one hand, the MRS increases rapidly during expansions, as the marginal utility of consumption relative to leisure quickly decreases, but on the other hand, there is no corresponding strongly pro-cyclical movement in real wages in the data. This gap between the MRS and the real wage, the so-called labor-market wedge, when treated as an exogenous inefficiency is an important source of economic fluctuations in this class of models; see Hall (1997) and Chari, Kehoe, and McGrattan (2007). Of course one would prefer to explain the wedge rather than treating it as an exogenous shock.[2]

Recently, Jaimovic, Rebelo, and Wong (2018) have documented that during the Great Recession, consumers reduced the quality of the goods and services they consumed. Since part of the labor wedge is due to the counter-cyclical marginal utility of consumption, pro-cyclical variation of quality can reduce the volatility of the labor wedge. While Jaimovic et al. (2018) provide a nice framework that includes quality substitution, the measurement of

[1] For example, Lucas and Rapping, 1969; Kydland and Prescott, 1982.

[2] The existing literature offers various interpretations for this wedge, including changes in home production technology (Benhabib, Rogerson, and Wright, 1991), government spending being a part of private consumption (Christiano and Eichenbaum, 1992), various frictions in the labor market, such as wage rigidity (Gali, Gertler, and Lopez-Salido, 2007) or search frictions (Shimer, 2010), and aggregation errors (Chang and Kim, 2007).

quality is very challenging. Instead, in this paper we study the average quality effects stemming from composition changes in the household's consumption basket and non-homothetic income-expenditure paths, that is, Engel curves. Observations on the shape of Engel curves are *easily obtained from* cross-sectional data such as the Consumer Expenditure Survey (CEX). Bils and Klenow (1998) have shown that these cross-sectional Engel curves can be good predictors of demand shifters over the business cycle.

We show that the substitution between necessities and luxuries can account for some of the cyclical movement of the labor-market wedge. In booms, households' consumption of luxuries (e.g., food away from home) tends to increase relatively more than the consumption of necessities (e.g., food at home). This substitution along the Engel curve slows down the increase in the MRS because the marginal utility of consumption falls more slowly as consumers switch toward luxuries. For a parameterization of non-homothetic Engel curves based on the cross-sectional household expenditure pattern across income quintiles in the CEX, we show that cyclical composition changes in the consumption basket can account for 6-15% of the volatility in the labor-market wedge measured in the aggregate time series data.

This note is organized as follows. Section 2 briefly discusses the measurement of the labor-market wedge and lays out a simple model where the household's preferences exhibit an Engel curve. In Section 3, based on the micro data that match the cross-sectional household expenditure pattern across income quintiles in the CEX, we compute the labor market wedge corrected for the Engel curve. Section 4 provides a concluding remark.

# 2 Labor-Market Wedge

To understand the role of the Engel curve in the measurement of the labor-market wedge, we first present the standard labor-market wedge for household preferences expressed with

respect to an aggregate consumption good $C$ and hours worked $H$:

$$\begin{aligned} U(C,H) &= \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \psi \frac{H^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \\ P \cdot C &= W \cdot H \end{aligned}$$

where $\sigma$ is the inter-temporal elasticity of substitution (IES) for consumption and $\gamma$ is the Frisch elasticity of labor supply.[3] The labor-market wedge $\tau$ is defined as the ratio between the marginal rate of substitution (between leisure and consumption) and the real wage $(W/P)$:

$$\frac{\psi H^{1/\gamma}}{C^{-1/\sigma}} = \frac{MU_L}{MU_C} = MRS = \tau \frac{W}{P}. \tag{1}$$

When we denote $\hat{x}$ for the cyclical component of $x$ (de-meaned growth rate or percentage deviation from the trend), the cyclical component of the labor-market wedge can be expressed as:

$$\hat{\tau} = \frac{1}{\gamma}\hat{H} + \frac{1}{\sigma}\hat{C} - \widehat{\frac{W}{P}} \tag{2}$$

Figure 1 shows the cyclical component of aggregate GDP and the labor-market wedge for a baseline parameterization of preferences using aggregate times series data. For a wide range of empirically plausible values of $\sigma$ and $\gamma$, the measured wedge is highly volatile and pro-cyclical because: (i) hours worked, $H$, and consumption, $C$, are both pro-cyclical, with $H$ being very volatile, and (ii) the real wage is neither highly pro-cyclical nor volatile. As shown in the table of Figure 1, (i) $H$ is slightly more volatile than GDP and highly pro-cyclical with a 0.95 elasticity with respect to GDP growth, and (ii) while $C$ and $W/P$ exhibit similar volatility the real wage is mildly pro-cyclical with a mere 0.19 elasticity with respect to GDP growth. As a result, the labor-market wedge defined for the baseline parameterization, $\sigma = 0.5$ and $\gamma = 1$, is tightly correlated with GDP and more than twice as volatile: a 1% increase in GDP is associated with a nearly 2% increase in the labor-market wedge.

[3]Since the labor-market wedge is entirely based on the intra-temporal optimality condition, we abstract from the dynamic decisions of households (e.g., savings, etc.).

We believe our baseline parameterization is plausible since (i) there is ample evidence of an inter-temporal elasticity of substitution in consumption much smaller than one, and (ii) variations in aggregate hours reflect the extensive margin as well as the intensive margins of labor (Keane and Rogerson (2012)).

Now, suppose that the household purchases $N$ types of consumption goods, $\{c_1, ..., c_N\}$, at prices $\{p_1, ..., p_N\}$. The household maximizes a utility function with inter-temporal elasticities of substitution that differ across goods

$$\begin{aligned} U(c_1, .., c_N, H) &= \sum_{i=1}^{N} \phi_i \frac{c_i^{1-1/\sigma_i}}{1-1/\sigma_i} - \psi \frac{H^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \\ P^m \cdot C^m &= \sum_{i=1}^{N} p_i c_i = W \cdot H \end{aligned}$$

where $P^m$ and $C^m$ represent the measured aggregate price and consumption index. The FOCs' are

$$\begin{aligned} \phi_i c_i^{-1/\sigma_i} &= \lambda p_i, \text{ for } i = 1, ..., N \\ \psi H^{1/\gamma} &= \lambda W \end{aligned} \tag{3}$$

where $\lambda$ is the marginal utility of nominal expenditures. This specification yields non-homothetic Engel curves across goods. A good with a small $\sigma_i$ is a necessity (e.g., food) whose marginal utility decreases rapidly with increased consumption. A good with a large $\sigma_i$ is a luxury whose marginal utility decreases slowly.

Summing over the FOCs for the consumption goods, we get the marginal utility of expenditures

$$\lambda = \frac{\sum_i \phi_i c_i^{1-\frac{1}{\sigma_i}}}{\sum_i p_i c_i} = \frac{\tilde{c}}{P^m \cdot C^m} \text{ with } \tilde{c} \equiv \sum_i \phi_i c_i^{1-\frac{1}{\sigma_i}}.$$

The *true* labor-market wedge, $\tau^*$, is then defined by the expression

$$\psi H^{1/\gamma}\frac{C^m}{\tilde{c}} = \frac{MU_L C^m}{\tilde{c}} = \tau^*\frac{W}{P^m}. \tag{4}$$

Compared to the standard measure of the labor-market wedge in (1) with aggregate consumption, this wedge with multiple goods is likely to be less cyclical because in economic booms households' consumption moves toward luxuries whose marginal utility decreases more slowly. The cyclical component (growth rate) of the labor-market wedge is[4]

$$\hat{\tau}^* = \frac{1}{\gamma}\hat{H} + C^m - \sum_i \left(1 - \frac{1}{\sigma_i}\right)\omega_i \hat{c}_i - \widehat{\frac{W}{P^m}}. \tag{5}$$

Suppose that the measured aggregate consumption and price indices are constructed as Divisia indices, $\hat{C}^m = \sum_i \omega_i \hat{c}_i$ and $\hat{P}^m = \sum_i \omega_i \hat{p}_i$, and are used in expression (2) to construct the measured wedge $\hat{\tau}^m$. Then the difference between the measured wedge and true wedge is simply:

$$\begin{aligned}\hat{\tau}^m - \hat{\tau}^* &= \sum_i \left(1 - \frac{1}{\sigma_i}\right)\omega_i \hat{c}_i - \left(1 - \frac{1}{\sigma}\right)\sum_{i=1}^{N}\omega_i \hat{c}_i \\ &= \sum_i \left(\frac{1}{\sigma} - \frac{1}{\sigma_i}\right)\omega_i \hat{c}_i.\end{aligned}$$

# 3 Empirical Analysis

## 3.1 Engel Curves from the CEX

We use 11 categories of household expenditures in the CEX: food at home, food away from home, transportation (excluding vehicle purchases), vehicle purchases, housing, utilities, health care, apparel, household operations, entertainment, and cash contribution. Table 1 shows their expenditure shares. We exclude "insurance and pensions" because they may

[4]We have used $\hat{\tilde{c}} = \sum_i \left(1 - \frac{1}{\sigma_i}\right)\omega_i \hat{c}_i$ where $\omega_i$ is the expenditure share of the $i$-th good.

reflect the household's savings rather than consumption. These 11 categories make up 84% of the total expenditure—close to 98% of the consumption-related expenditure (total expenditure net of those on personal insurance and pensions).

For each consumption category $i$, the Engel curve parameter, $\sigma_i$, can be estimated as follows. The FOCs of the household's utility maximization for consumption goods (3) imply that for any two goods

$$\ln c_i = \frac{\sigma_i}{\sigma_j} \ln c_j - \sigma_i \ln(p_i/p_j). \tag{6}$$

Let $c_i^{Qk}$ denote the quantity of consumption for category $i$ by the household in the $k$-th quintile of the income distribution. Assuming that households face the same prices we get

$$\ln \left( \frac{c_i^{Q5}}{c_i^{Q1}} \right) = \frac{\sigma_i}{\sigma_j} \ln \left( \frac{c_j^{Q5}}{c_j^{Q1}} \right) \tag{7}$$

and we can infer the *relative* Engel curves between categories $i$ and $j$, $\sigma_i/\sigma_j$, from the cross-sectional consumption ratios of the respective categories for households in the 5th and 1st income quintile. We normalize the average of the Engel curve parameters $\sigma$ to be 0.5.[5] Based on the cross-sectional CEX of 2015, we compute the relative (to total expenditure) Engel curve $\sigma_i/\sigma$'s, according to (7) and report in the last column of Table 1.

For the price of each consumption category, we use the corresponding price index from the CPI. For "Entertainment" and "Cash Contribution," for which the CPI does not have a separate price index, we use the aggregate CPI. It is well-known that aggregate expenditures in the CEX are growing at a much slower pace than aggregate PCE in the NIPA because the CEX systematically understates certain items in households' expenditures. Figure 2 shows that aggregate PCE increased 2.3 times from 1985 to 2015, whereas aggregate CEX expenditures do not exhibit a strong upward trend. Since we focus on the cyclical components of consumption, this is less of a concern for our analysis.[6] Indeed, Figure 3 shows that the

[5]That is, the Engel curve of total consumption expenditure is 0.5.

[6]This discrepancy between the CEX and the PCE can of course create a serious problem for the analysis of time trends. See, e.g., Aguiar and Bils (2015), on the analysis of trends in income and consumption inequality.

cyclical components (de-meaned growth rates) of aggregated consumption measures comove fairly closely.

## 3.2 Cyclical Behavior of Labor-Market Wedges

We first show that the volatility of the labor wedge constructed with our aggregate measure of consumption from the CEX is comparable with that of labor wedges constructed from more standard measures of aggregate consumption. We then show that the labor wedge constructed from the disaggregated CEX categories is less volatile than the labor wedge from the CEX aggregate.

The first three rows of Table 2 display the volatility of the labor-market wedge based on the standard single-goods utility for three measures of aggregate consumption: all items of PCE in the NIPA, nondurables and services of PCE, and the sum of our 11 CEX expenditure categories, "CEX11-Aggregate." Since our framework applies to nondurable goods, the PCE for nondurable goods and services is the appropriate aggregate consumption measure. The labor wedge from the CEX aggregate is slightly less volatile than the nondurable goods and services of the PCE from the NIPA.

We now compare the two labor wedges using two different measures of the marginal utility of consumption based on 11 CEX consumption categories: the aggregated consumption ("CEX11-Aggregate") versus the disaggregated Engel-curve-based ("CEX11-Engel") measure. From the third and fourth rows of Table 2 we can see that accounting for differences in income elasticities across commodities reduces the volatility of the labor-market wedge by about 9.3%. In other words, recognizing the differences in marginal utility across commodities together with the pro/counter-cyclical nature of luxuries/necessities makes true marginal utility move less than is implied by the usual aggregate consumption measure and results in a less volatile labor-market wedge.[7]

[7]We have also computed the wedge excluding "vehicle purchase" because vehicles are durable goods. In this case, the cyclicality of the wedges decreases by 6% from 1.45 for CEX11-Aggregate and 1.37 for CEX11-Engel.

Table 2 also reports the volatility of the labor-market wedge based on alternative values of $\sigma$ and $\gamma$. Using a smaller inter-temporal elasticity of consumption magnifies the labor-market wedge– it is even harder to justify the cyclical behavior of consumption and hours as an optimal choice of the stand-in household. With $\sigma = 0.1$, the volatility based on the CEX11-Aggregate increases to 5.09–the wedge moves five times as much as GDP over the business cycle. The volatility of the "true" wedge (CEX11-Engel) is 4.33, roughly 15% smaller than the standard measure. Using the larger value $\sigma = 1$, that is, log utility in consumption, our treatment reduces the wedge volatility by only 6.3%. A larger labor supply elasticity reduces the volatility of the wedge because the marginal utility of leisure increases at a slower rate in booms. The same reduction in the volatility of the marginal utility of consumption from using disaggregated Engel curves then implies a larger percentage reduction in the labor wedge volatility. Overall, correcting the movement of the marginal utility of consumption based on the differences in the Engel curve across 10 consumption categories in the CEX decreases the volatility of the wedge by 6-15%; see row (6) of Table 2.

We obtain an upper bound on how much one can reduce the labor-market wedge by correcting the measurement of the marginal utility of consumption by making the marginal utility of consumption a constant, $\sigma = \infty$; see row (5) of Table 2. From equation (1) it follows that this specification provides an upper bound because consumption and labor supply are positively correlated and the real wage is essentially acyclical. For example, with $\gamma = 1$ and $\sigma = 0.5$, assuming a constant marginal utility of consumption reduces the estimated volatility of the wedge by half relative to the benchmark case. Our treatment based on non-homothetic Engel curves across 11 categories in the CEX materialize 18% of this potential reduction in the volatility of wedge. Note also that the *relative* contribution of our correction of the wedge remains at 18% regardless of $\gamma$'s and $\sigma$'s; see row (8) of Table 2.

# 4 Concluding Remark

Estimated equilibrium models of the business cycle have been widely used to study the effects of government policies over the business cycle, e.g., Smets and Wouters, 2007; Christiano, Eichenbaum, and Evans, 2005. One popular way to identify the sources of economic fluctuations in these models is to measure shocks as 'distortions' in a model-implied relationship among key aggregate time series, e.g., an optimality condition or a resource constraint. According to this method, the labor-market wedge—the gap between the MRS between consumption and leisure and the real wage—often emerges as an important source of aggregate fluctuations.

Jaimovic et al. (2018) show that trading down toward low-quality goods and services during recessions can generate a counter-cyclical labor-market wedge. While this mechanism provides a potentially important mechanism to fill the gap between the model and the data, the quality of goods is not easy to quantify. In this note, we study changes in the average quality of aggregate consumption based on observable changes in the composition of households' consumption baskets when household Engel curves are non-homothetic. For example, in booms households increase the relative consumption of commodities that one might consider luxury goods and whose marginal utility decreases more slowly with increased consumption. Ignoring this cyclical substitution within the consumption basket makes the wedge between the MRS and real wages appear larger than it actually is. When we take into account the differences in the Engel curve across 11 consumption categories in the CEX, the volatility of the measured labor-market wedge decreases by 6-15%.

# References


[1] **Aguiar, Mark and Mark Bils.** 2015. "Has Consumption Inequality Mirrored Income Inequality?" *American Economic Review*, 105, 2725-2756.

[2] **Benhabib, Jess, Richard Rogerson, and Randall Wright.** 1991. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations." *Journal of Political Economy*, 99(6): 1166-1187.

[3] **Bils, Mark, and Peter J. Klenow.** 1998. "Using Consumer Theory to Test Competing Business Cycle Models." *Journal of Political Economy*, 106(2): 233-261.

[4] **Chang, Yongsung, and Sun-Bin Kim.** 2007. "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations." *American Economic Review*, 97(5), 1939-1956.

[5] **Chari, Varadarajan V., Patrick J. Kehoe, and Ellen R. McGrattan.** 2007. "Business Cycle Accounting." *Econometrica*, 75, 781-836.

[6] **Christiano, Lawrence J., and Martin Eichenbaum.** 1992. "Current Real-business cycle Theories and Aggregate Labor-Market Fluctuations." *American Economic Review*, 82(3): 430-450.

[7] **Christiano, Lawrence, Martin Eichenbaum, and Charles Evans** 2005. "Nominal Rigidities and the Dynamic Effect of a Shock to Monetary Policy." *Journal of Political Economy*, 113, 1-45.

[8] **Gali, Jordi, Mark Gertler, J. David Lopéz-Salido.** 2007. "Markups, Gaps, and the Welfare Costs of Business Fluctuations." *Review of Economics and Statistics*, 89(1), pages 44-59.

[9] **Hall, Robert E.** 1997. "Macroeconomic Fluctuations and the Allocation of Time." *Journal of Labor Economics*, 15(1): s223-s250.

[10] **Jaimovic, Nir, Sergio Rebelo and Arlene Wong.** 2018. "Trading Down and the Business Cycle." *Journal of Monetary Economics*, forthcoming.

[11] **Keane, Michael, and Richard Rogerson.** 2012. "Micro and Macro Labor Supply Elasticities: A Reassessment of Conventional Wisdom." *Journal of Economic Literature*, 50(2): 464-476.

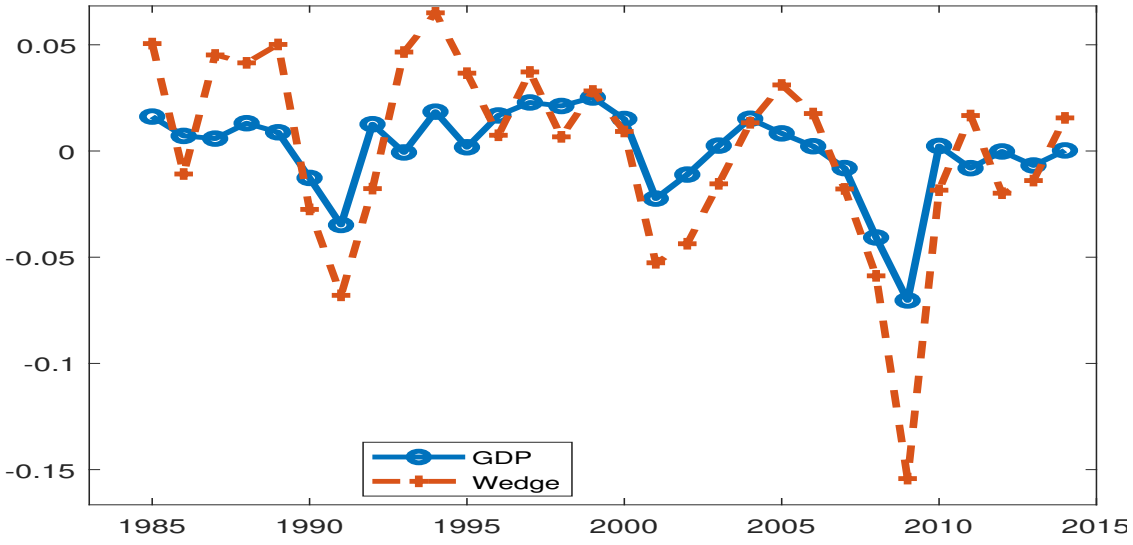
[12] **Kydland, Finn, and Edward Prescott**. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica*, 50(6): 1345-1370.

[13] **Lucas, Robert E., Jr., and Leonard A. Rapping.** 1969. "Real Wages, Employment, and Inflation." *Journal of Political Economy*, 77(5): 721-754.

[14] **Shimer, Robert.** 2010. *Labor Markets and Business Cycles*. Princeton University Press.

[15] **Smets, Frank, and Rafael Wouters.** 2007. "Shocks and Frictions in U.S. Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97, 586-606.

Figure 1: Cyclical Behavior of the Labor-Market Wedge




| | GDP | H | C | W/P | Wedge ($\tau$) |
|---|---|---|---|---|---|
| SD (%) | 2.06 | 2.34 | 1.31 | 1.50 | 4.55 |
| Cyclicality | 1 | 0.95 | 0.56 | 0.19 | 1.88 |

Note: The labor-market wedge is computed under $\sigma = 0.5$ and $\gamma = 1$. The aggregate consumption ($C$) and its price are based on personal consumption expenditures (PCE) on nondurables and services from the NIPA. Aggregate hours ($H$) and nominal wages ($W$) are total hours and wages from the BLS's Labor Productivity and Cost index (LPC). All variables are growth rates (first differences in logs). SD denotes the standard deviation. "Cyclicality" denotes the regression coefficient on GDP growth.

Figure 2: Aggregate Consumption

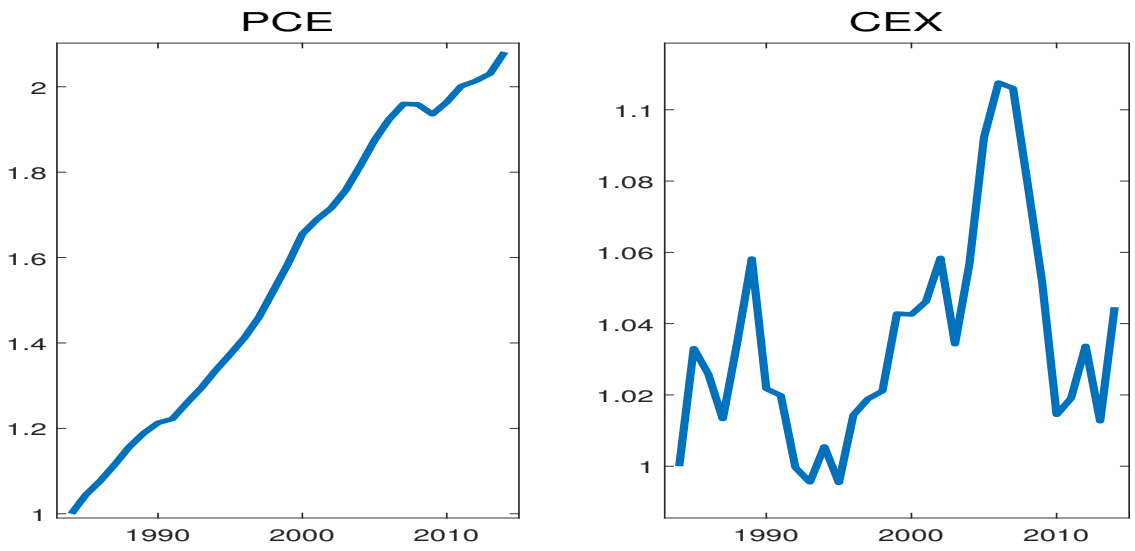



Figure 3: Cyclical Components of Consumption

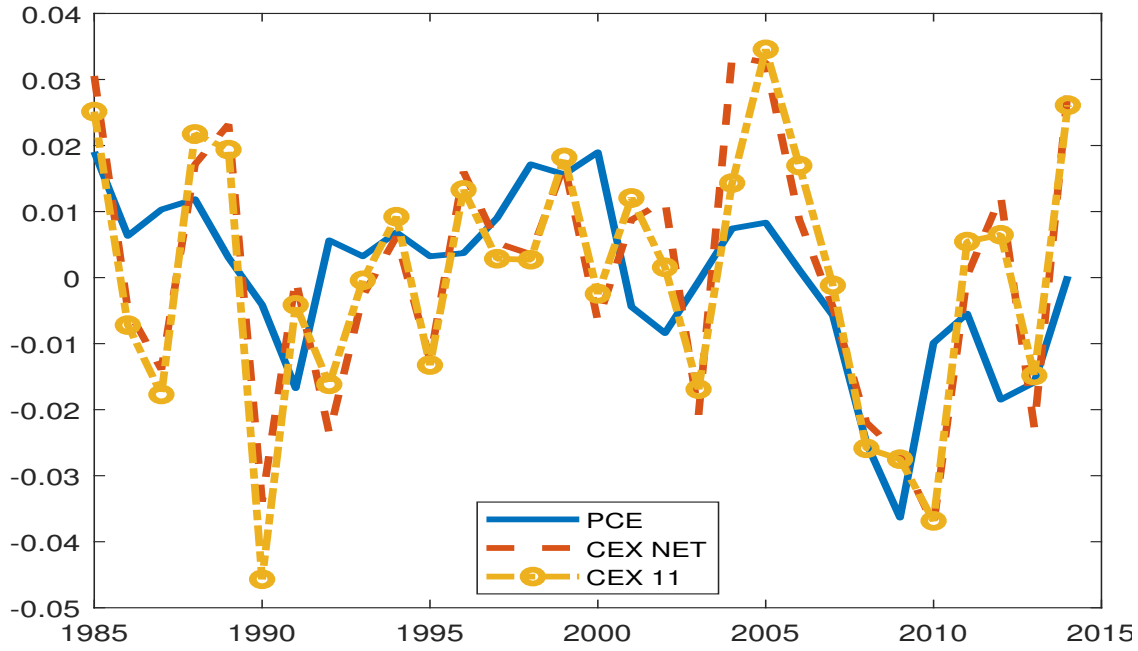


Table 1: Relative Engel Curves

| Category | Share | Relative Engel ($\frac{\sigma_i}{\sigma}$) |
|---|---|---|
| Food at Home | 7.2% | 0.68 |
| Food away from Home | 5.4% | 1.15 |
| Transportation | 17 % | 1.18 |
| (Vehicle Purchase) | (7.1%) | 1.42 |
| Housing | 32.9% | 0.89 |
| Health Care | 7.8% | 0.95 |
| Utilities | 6.8% | 0.65 |
| Apparel | 3.3% | 1.12 |
| Household Operations | 2.3% | 1.37 |
| Entertainment | 5.1% | 1.13 |
| Cash Contribution | 3.2% | 1.29 |
| Sum of 11 Categories (**CEX11**) | **84.2**% | – |
| Other Misc. | 4.5% | – |
| Sum of All above (**CEX NET**) | **88.7**% | – |
| Personal Insurance and Pension | 11.3% | 2.5 |
| All Items | 100% | 1 |

Note: The data are based on the annual expenditure means and shares by quintiles of income before taxes from the Consumer Expenditure Survey 2015 (Table 1101).

Table 2: Cyclicality of Labor Wedges

| | Measure for Marginal Utility of C | $\sigma = 0.5$ $\gamma = 1$ | $\sigma = 0.1$ $\gamma = 1$ | $\sigma = 1$ $\gamma = 1$ | $\sigma = 0.5$ $\gamma = 2$ | $\sigma = 0.5$ $\gamma = 0.5$ |
|---|---|---|---|---|---|---|
| (1) | PCE (all) | 2.15 | 7.71 | 1.46 | 1.68 | 3.10 |
| (2) | PCE (nondurables + services) | 1.88 | 6.35 | 1.32 | 1.41 | 2.83 |
| (3) | **CEX11-Aggregate** | **1.63** | **5.09** | **1.19** | **1.15** | **2.58** |
| (4) | **CEX11-Engel** | **1.48** | **4.33** | **1.12** | **1.00** | **2.42** |
| (5) | Constant $MU_C$ ($\sigma = \infty$) | 0.76 | 0.76 | 0.76 | 0.29 | 1.71 |
| (6) | $\frac{(4)}{(3)} - 1$ | -9.3% | -14.8% | -6.3% | -13.1% | -5.9% |
| (7) | $\frac{(5)}{(3)} - 1$ | -53% | -85% | -36% | -75% | -34% |
| (8) | $\frac{(6)}{(7)}$ | 18% | 18% | 18% | 18% | 18% |

Note: PCE is personal consumption expenditures from the NIPA. CEX11 is based on the 11 categories in the CEX. When $\sigma$ is close to infinity, the marginal utility of commodity consumption ($MU_C$) becomes a constant, where the labor-market wedge is entirely driven by the marginal utility of leisure (relative to real wage).