On the profitability of interfirm bundling in oligopolies

Sang-Hyun Kim*and Jong-Hee Hahn†

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Abstract

This paper examines the profitability of bundling or exclusive dealing among independent firms selling differentiated products. We show that, compared with separate sales, interfirm bundling generally raises prices and is more profitable provided the distribution of consumer valuations for products are sufficiently symmetric and centered in the middle. Hence the firms have mutual incentives to offer their products as a bundle or make exclusive dealing arrangements. We shed new light on the role of bundling in relaxing competition in oligopoly, the importance of which has been neglected in the previous literatures.

JEL Classifications: L11, L13

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*School of Economics, Yonsei University, Seoul 03722, South Korea. E-mail: sang.kim@yonsei.ac.kr

†Corresponding author: School of Economics, Yonsei University, Seoul 03722, South Korea. E-mail: hahnjh@gmail.com

1 Introduction

In reality we often find different products offered by independent sellers being sold as bundles. It takes various forms such as interfirm bundling, exclusive dealing, incompatibility, technological alliances, brand-specific discounts, and so on.[1] A typical example is a closed system of hardware and software. Some PC software applications are compatible only with a specific operating system such as MS Windows or Mac OS. Similarly, some mobile applications are available only on a certain operating system such as Android or iOS. Also, some video games work only in a specific game console. Some mobile phones can be purchased only from a specific network operator. Other examples include strategic alliances on technical standards, which go back to the historic case of VHS versus Beta in video recording format and more recently mobile protocols used in second-generation (2G) wireless communications, CDMA (code division multiple access) versus GSM (global system for mobiles). Examples are found in the consumption goods industry as well. Some family restaurants carry only a specific brand of cola, while others carry only the rival's brand. Also relevant is brand-specific discounts where a firm offers a price discount conditional on the purchase of another firm's product: credit card companies give a price discount to customers who purchase from a specific shop, petrol station, hotel chain, car rental company, amusement part, and so on (see Gans and King (2006) and Hahn and Kim (2016) for more examples). Regardless of whether it is physical or contractual, such exclusive arrangements raise policy concerns since they could limit consumer choice and thus restrain market competition.

There is a rich economic literature on product bundling. Early research mainly focused on the profitability of bundling and the optimal pricing of bundles in the context of a multi-product monopolist's price discrimination.[2] Since the seminal paper by Whin-

[1] If brand-specific discounts are sufficiently large, consumers are in effect forced to choose one of bundled packages.

[2] See Stigler (1968), Adams and Yellen (1976), Schmalensee (1984), Long (1984), McAfee et al.

ston (1990), attention has been given to anti-competitive foreclosure effects of bundling.[3] Some authors studied a multi-product firm's incentive to bundle their products for the purpose of relaxing competition (see Carbajo et al. (1990), Chen (1997) and Denicolo (2000)). Armstrong (2013) has shown that independent sellers of two substitutes may wish to jointly offer interfirm discounts in order to relax competition.

The literature on competitive bundling in oligopolies is relatively small. Based on the two-dimensional Hotelling model, Matutes and Regibeau (1988) analyzed firms' incentives for mix-and-match compatibility in system markets. They showed that two integrated firms supplying both components wish to make their products compatible with the rival firm's components. This is because with compatible components a firm cannot internalize the full benefit of its price cut and thus price competition is softened compared with under incompatibility where the same price cut results in a demand increase in both components.[4] This imperfect appropriation (under compatibility) or double profitability (under incompatibility) effect, however, disappears when the market consists of four independent suppliers each offering a single component, and in this case the firms' profits are the same regardless of compatibility or incompatibility. A similar result has also been discovered by Gans and King (2006) in the context of interfirm bundled discounts.

Since then a line of research has noticed that the density of equilibrium market boundary is also important in determining the price effect of competitive bundling, and showed that the result can be reversed due to this market boundary effect. Asymmetry

(1989), Fang and Norman (2006), Armstrong and Vickers (2010), Chen and Riordan (2013), Armstrong (2013), and so on.

[3]Subsequent researches include Choi and Stefanadis (2001), Carlton and Waldman (2002), and Nalebuff (2004).

[4]Einhorn (1992) established a similar result in the context of vertical differentiation, where compatibility softens competition by increasing the degree of quality differentiation. See also Matutes and Regibeau (1992) and Thanassoulis (2007) for an analysis on the case where each firm can sell an individual components separately together with the bundle.

in cost or quality across firms can critically affect the incentive for bundling by changing the relative market boundary under the two regimes, as shown by Hahn and Kim (2012) and Hurkens et al. (2016). Hermalin and Katz (2013) found that if the degree of product differentiation differs across products Matutes-Regibeau's result can be reversed. Similarly, Hahn and Kim (2016) showed that two independent firms, one competing in a market for homogeneous goods and the other competing in a market for differentiated goods, have incentives to jointly offer bundled discounts. Also, using a random utility framework Zhou (2017) showed that the bundling incentive critically depends on the number of firms in the markets, because the mass of the market boundary gets smaller under bundling as the number of firms increases.

Although the previous works provided us with useful insights on the competitive effect of oligopoly bundling, it seems premature to assert that we fully understand the mechanism how interfirm bundling affects equilibrium prices, profits, and consumer welfare. In this paper, we distinguish two channels by which interfirm bundling affects equilibrium price and profits. The one is the mass of indifferent consumers, which we call the density of equilibrium market boundary, that critically depends on the shape of the joint distribution of consumer valuations for the products. Naturally, as the mass of consumers at the margin becomes larger, the intensify of price competition increases. The other is the difficulty of consumers' brand switching due to interfirm bundling. It is more costly for a consumer to switch brand when products are sold as bundles rather than individual components. This paper attempts to disentangle the switching-hindering effect, which has been somewhat neglected in the exiting literature, from the market boundary effect, and highlights its role in determining the profitability of interfirm bundling in oligopoly.

Using copulas to model the dependence of values for products we derive a sufficient condition on the distribution of consumer valuations, under which interfirm bundling is profitable for the firms.[5] Specifically, we show that interfirm bundling raises prices and profits, provided the valuation distribution is sufficiently symmetric and its marginal density is quasi-concave. In such cases, the market boundary effect is neutralized and the switching-hindering effect stands out as a dominant factor. As a result, equilibrium prices are higher under interfirm building relative to separate sales.

We find that those two effects are exactly canceled out when consumers are distributed uniformly on the unit square, as in Matutes and Regibeau (1988) and Gans and King (2006). Our analysis reveals that this profit-neutrality result holds only in the knife-edge case of uniform distribution on the unit square. Here the mass of consumers on the diagonal line (the market boundary under bundling) is greater than those on the horizontal or vertical line (the market boundary under separate sales). Interestingly, in this case the negative market boundary effect exactly cancels out the positive switching hindering effect of bundling. If the mass on the diagonal line is reduced even slightly, the firms would strictly prefer interfirm bundling to separate sales. For example, if consumers are uniformly distributed on a disk, interfirm bundling will be strictly more profitable than separate sales.

In sum, our analysis sheds new light on the firms' incentives for interfirm bundling. We distinguish the switching-hindering effect as one of main factors determining the competitive effect of bundling among independent firms, and derive some general conditions under which firms have mutual incentives to sell their products as bundles.

[5] Recently there have been a few studies using the copula representation of consumer preferences in analyzing pricing problems. Chen and Riordan (2013, 2015) applied the copula technique to analyze the profitability of mixed bundling by a multiproduct monopolist and the relationship between preference dependence and market outcomes in symmetric multiproduct industries. To our knowledge, ours is the first analysis using the copula technique in the Hotelling framework.

2 Model

Consider markets for two independent products, each supplied by two horizontally differentiated suppliers. A continuum of consumers with unit demands for each product are distributed in the unit square according to a joint distribution function $G(x_1, x_2)$, where (x_1, x_2) denotes consumer location in the product space $[0,1]^2$. The joint density function is denoted by $g(x_1, x_2) = \partial^2 G(x_1, x_2)/\partial x_1 \partial x_2$. Let $F_i(x_i)$, $i = 1, 2$, denote the marginal distribution derived from $G(x_1, x_2)$. We focus on the case where $G(x_1, x_2)$ is symmetric, i.e., $G(x_1, x_2) = G(x_2, x_1)$, which means the marginal distribution is identical, i.e., $F_1(x) = F_2(x) = F(x)$. We further assume that the marginal density $f(x)$ is quasi-concave and symmetric across the mid point $x = 1/2$. This implies that $f(1/2) \geq f(x_i)$ for all $x_i \in [0,1]$. That is, there are no fewer consumers in the middle than any other locations in the product space. Define $k(a) = \int_0^1 g(x, a-x)dx$ and $\overline{k}(a) = \int_0^1 g(x, x-1+a)dx$, which denote the densities of consumers on the diagonal lines for a given a, i,e, the densities of the market boundaries when the products are bundled. To ensure the existence of an interior equilibrium under interfirm bundling, we assume that both $k(a)$ and $\overline{k}(a)$ are quasi-concave for $a \in [0,2]$.

One of the firms selling product i is located at 0 and the other is at 1 on ith coordinate, $i = 1, 2$. Let us denote the seller at location 0 firm iA and the other at location 1 firm iB. For simplicity, we assume that production costs are zero for all firms. If a consumer located at x_i on ith coordinate buys product i from firm ij ($j = A, B$), her utility is given by $v_i - t\,|x_i - x_j| - p_{ij}$, where v_i is the gross value of the product, t is the transportation cost per unit of distance, $x_j \in \{0,1\}$ is the location of firm ij and p_{ij} is the price she pays. We assume that v_i is large enough for both products so that the markets are fully covered in equilibrium.[6] Then the total utility a consumer at (x_1, x_2)

[6] Under this assumption, we can treat two products as perfectly complementary components which are essential to make a whole system.

gains when buying product 1 from firm $1j$ and product 2 from firm $2k$ is given by

$$U(x_1, x_2) = v_1 - t|x_1 - x_j| - p_{1j} + v_2 - t|x_2 - x_k| - p_{2k}, j, k = A, B.$$

In contrast to the previous models on interfirm bundling or bundled discounts, we allow for positive or negative correlations in consumer preferences for two products. Specifically, we model the dependence of consumer valuations for two products using a copula. A copula is a multivariate probability distribution for which the marginal probability distribution of each variable is uniform.

Let us denote the marginal distribution of random variable X_i as $Y_i \equiv F_i(x_i)$, $i = 1, 2$. Then the copula of (x_1, x_2) is defined as the joint cumulative distribution function of (Y_1, Y_2):

$$C(y_1, y_2) = \Pr[Y_1 \leq y_1, Y_2 \leq y_2].$$

That is, the copula $C(y_1, y_2)$ is a bivariate uniform distribution that "couples" two marginal distributions Y_1 and Y_2 in order to describe the dependence between the original random variables X_1 and X_2.[7] According to Sklar's theorem, any joint distribution of random variables can be represented by marginal distribution functions and a copula which describes the dependence structure between the random variables. So a copula C must exist such that $G(x_1, x_2) = C(F(x_1), F(x_2))$. Since a copula is a multivariate uniform distribution, $C(y_1, 1) = \Pr(Y_1 \leq y_1, Y_2 \leq 1) = \Pr(Y_1 \leq y_1) = y_1$, and $C(y_1, 0) = \Pr(Y_1 \leq y_1, Y_2 \leq 0) = 0$. Moreover, the partial derivative $C_1(y_1, y_2) = \partial C(y_1, y_2)/\partial y_1$ is the conditional distribution of y_2 given $Y_1 = y_1$, and the cross-partial derivative $C_{12}(y_1, y_2) = \partial^2 C(y_1, y_2)/\partial y_1 \partial y_2$ is the joint density function.

Since we are considering symmetric joint distributions, the corresponding copulas are also symmetric, i.e., $C(y_1, y_2) = C(y_2, y_1)$. Assume that there exists a single parameter $\theta \in [-\overline{\theta}, \overline{\theta}]$ which governs the dependence of Y_1 and Y_2 in the following ways:

[7] It would be straightforward to extend our analysis to the case of more than two products using a multivariate copula.

A1. For $\theta = 0$, $C_{12}(y_1, y_2; 0) = 1$ for all $y_1, y_2 \in [0, 1]$.

A2. For $\theta > 0$, $C_{12}(y, 1-y; \theta) \leq 1 \leq C_{12}(y, y; \theta)$ for all $y \in [0, 1]$, and strict inequalities hold for some y.

A3. For $\theta < 0$, $C_{12}(y, y; \theta) \leq 1 \leq C_{12}(y, 1-y; \theta)$ for all $y \in [0, 1]$, and strict inequalities hold for some y.

These assumptions say that Y_1 and Y_2 are independent for $\theta = 0$, positively correlated for $\theta > 0$, and negatively correlated when $\theta < 0$. That is, the density on the 45 degree line (i.e., $y_2 = y_1$) is higher(lower) than that on $y_2 = 1 - y_1$ line when θ is positive(negative). An example of such copulas is Fairlie-Gumbel-Morgenster (FGM) copula:

$$C(y_1, y_2; \theta) = y_1 y_2 \left[1 + \theta(1 - y_1)(1 - y_2)\right]$$

for $\theta \in [-1, 1]$. Note that $C_{12}(y_1, y_2; \theta) = 1 + \theta(2y_1 - 1)(2y_2 - 1)$, and so $C_{12}(y, y; \theta) = C_{12}(y, 1-y; -\theta) = 1 + \theta(2y - 1)^2$ which is greater than or equal to 1 for all y if and only if $\theta > 0$. We make these assumptions for the purpose of expositional convenience. Our main result (Proposition 1) would hold for more general classes of copulas with multiple parameters.

3 Separate sales versus Interfirm bundling

SEPARATE SALES: If the firms in each market sell their product separately without any relations to the other products sold in the other market, consumers' purchasing decisions are independent across two markets and competition in each market follows the standard one-dimensional Hotelling framework. A consumer at location x_i buys product i from firm iA if $tx_i + p_{iA} \leq t\,(1 - x_i) + p_{iB}$, and buys from firm iB otherwise. So, the profit of firm iA is given by

$$\pi_{iA}(p_{iA}, p_{iB}) = p_{iA} F\left(\frac{1}{2} - \frac{p_{iA} - p_{iB}}{2t}\right), i = 1, 2.$$

The first-order condition is

$$F\left(\frac{1}{2}-\frac{p_{iA}-p_{iB}}{2t}\right)-\frac{p_{iA}}{2t}f\left(\frac{1}{2}-\frac{p_{iA}-p_{iB}}{2t}\right)=0.$$

Imposing symmetry yields the following equilibrium price and profit:

$$p^S=\frac{t}{f(1/2)};\ \pi^S=\frac{t}{2f(1/2)}.$$

INTERFIRM BUNDLING: If two firms, say $1A$ and $2A$, jointly decided to sell their products as a bundle and commit not to sell those products separately, consumers have to choose one of two bundles, AA offered by firms $1A$ and $2A$ or BB offered by firms $1B$ and $2B$. It is as if each pair of two firms forms an alliance making their products incompatible with the others offered by the firms belonging to the other alliance. Then a consumer at location (x_1,x_2) will buy bundle AA if $tx_1+tx_2+p_{1A}+p_{2A}\le t(1-x_1)+t(1-x_2)+p_{1B}+p_{2B}$, i.e.,

$$x_2\le 1-x_1-\frac{P_A-P_B}{2t}, \tag{1}$$

and buy bundle BB otherwise, where $P_j=p_{1j}+p_{2j}$. Using $y_i\equiv F(x_i)$, we can rewrite (1) as

$$y_2\le F\left(1-F^{-1}(y_1)-\frac{P_A-P_B}{2t}\right).$$

Recall that $C_1(y_1,y_2)$ is the conditional distribution of y_2 given y_1. Assuming $P_A\ge P_B$, the demand for bundle AA can be written as

$$D_A(P_A,P_B)=\int_0^{F\left(1-\frac{P_A-P_B}{2t}\right)}C_1\left(y_1,F\left(1-F^{-1}(y_1)-\frac{P_A-P_B}{2t}\right)\right)dy_1.$$

Then the profit of firm $1A$ is given by $\pi_{1A}(p_{1A},p_{2A},P_B)=p_{1A}D_A(p_{1A}+p_{2A},P_B)$. Solving for a symmetric Nash equilibrium yields the following result.

Lemma 1 *The equilibrium price for each product and each firm's profit under interfirm bundling are given by*

$$p^B=\frac{t}{\Psi};\ \pi^B=\frac{t}{2\Psi}, \tag{2}$$

where $\Psi\equiv\int_0^1 C_{12}(F(x),1-F(x))f(1-x)f(x)dx$.

Proof. Differentiating D_A with respect to p_{iA} and imposing symmetry, we obtain the following:

$$\begin{aligned}
\left.\frac{\partial D_A}{\partial p_{iA}}\right|_{P_A=P_B} &= -\frac{1}{2t}\left[\int_0^1 C_{12}\left(y_1, F\left(1-F^{-1}(y_1)\right)\right) f\left(1-F^{-1}(y_1)\right) dy_1 + C_1(F(1),F(0))f(1)\right] \\
&= -\frac{1}{2t}\int_0^1 C_{12}\left(y_1, 1-y_1\right) f\left(1-F^{-1}(y_1)\right) dy_1,
\end{aligned}$$

where the use has been made of the facts that $F(1-F^{-1}(y_1)) = F(F^{-1}(1-y_1)) = 1-y_1$ given the symmetry of $f(y)$ across $1/2$ and $C_1(F(1),F(0)) = C_1(1,0) = 0$. Since $y_1 = F(x_1)$ and so $dy_1 = f(x_1)dx_1$, it must be that

$$\begin{aligned}
&\int_0^1 C_{12}\left(y_1, 1-y_1\right) f\left(1-F^{-1}(y_1)\right) dy_1 \\
&= \int_0^1 C_{12}\left(F(x_1), 1-F(x_1)\right) f\left(1-x_1\right) f(x_1)dx_1 \equiv \Psi.
\end{aligned}$$

So we obtain

$$\left.\frac{\partial D_A}{\partial p_{iA}}\right|_{P_A=P_B} = -\frac{\Psi}{2t}.$$

Plugging this into the first-order condition completes the proof:

$$\begin{aligned}
\frac{1}{2} + p_{1A}\left.\frac{\partial D_A}{\partial p_{iA}}\right|_{P_A=P_B} &= 0 \\
&\Longrightarrow p^B = -\frac{1}{2\left.\frac{\partial D_A}{\partial p_{iA}}\right|_{P_A=P_B}} = \frac{t}{\Psi}
\end{aligned}$$

■

The equilibrium price balances the infra-marginal and marginal effects of a small increase in price, which largely depends on the price elasticity of demand at the market boundary, i.e., $\partial D_A/\partial p_{iA}|_{P_A=P_B} = -\Psi/2t$. Note that the term Ψ is proportional to the mass of consumers at the market boundary depicted by the diagonal line $x_2 = 1 - x_1$.

Comparison of profits: First, consider the case where consumer valuations of two products are independent, i.e., $C_{12}(y_1, y_2) = 1$ for any (y_1, y_2). Then it holds that

$\Psi \equiv \int_0^1 f(x)f(1-x)dx = E[f(1-x)] \le f(1/2)$ under the quasi-concavity of $f(y)$ and its symmetry across $1/2$. This implies that if consumers have independent preferences for two products (i.e., $\theta = 0$), individual firms' profit under interfirm bundling is no smaller than the one obtained under separate sales ($\pi^B \ge \pi^S$), where the equality holds only when the consumer valuation for each product X_i is uniformly distributed.

Next, consider the cases where the valuations of two products are correlated across consumers, i.e., X_1 and X_2 are dependent. Suppose first that the two random variables are positively correlated, i.e., $\theta > 0$. A2 states that $C_{12}(y, 1-y; \theta) \le 1$ for all $y \in [0,1]$ and the strict inequality holds for some y, and thus it holds that

$$\begin{aligned}\Psi &= \int_0^1 C_{12}(F(x), 1-F(x))f(1-x)f(x)dx \\ &< \int_0^1 f(x)f(1-x)dx = E[f(1-x)] \le f(1/2).\end{aligned}$$

Intuitively, as X_1 and X_2 are more positively correlated, the mass of the indifferent consumers on the market boundary (represented by the line $x_2 = 1 - x_1$) becomes smaller, making price competition less intense under interfirm bundling relative to the case of separate sales.

On the other hand, if X_1 and X_2 are negatively correlated, it holds that $\Psi = \int_0^1 C_{12}(F(x), 1-F(x))f(1-x)f(x)dx > f(1/2)$. This corresponds to the case where the mass of consumers on the market boundary $x_2 = 1-x_1$ is sufficiently large, so that competition is more intense under interfirm bundling compared with separate sales. In such a case, however, firm $1A$ could bundle its product with firm $2B$'s product instead of $2A$'s. Then, the equilibrium price would be t/Ψ', where $\Psi' \equiv \int_0^1 C_{12}(F(x), F(x))f(x)^2 dx$. According to A3, $C_{12}(y, y; \theta) \le 1$ for all y and the strict inequality holds for some y. So it must be that $\Psi' < f(1/2)$, which means that the equilibrium price and each firm's profit are larger under interfirm bundling relative to separate sales. The discussion so far yields the following proposition.

Proposition 1 *Suppose that f is quasi-concave and symmetric. Then, the equilibrium* *price and profit under interfirm bundling is larger than or equal to those under separate sales, where the equality holds only when the valuations for two products are independent and the marginal distribution is uniform.*

The above result, although obtained under the assumption of symmetric and quasi-concave distribution of consumer valuations, would continue to hold in other situations, as long as the valuation distribution is sufficiently symmetric and concave. If, in contrast, the density function is convex, i.e., there are fewer consumers at the center than at the tails, then first of all, symmetric equilibria may not exist. Moreover, if it exists interfirm bundling would lead to more intense price competition, resulting in a reduction in profit for all the firms.

The next proposition presents a case where the profitability of interfirm bundling increases as the dependency of consumer valuations for two products becomes stronger.

Proposition 2 *Suppose that $C(y_1, y_2; \theta)$ is Fairlie-Gumbel-Morgenster (FGM) copula. Then, $\pi^B - \pi^S$ becomes greater as $|\theta|$ increases.*

Proof. Since $C(y_1, y_2; \theta)$ is FGM copula, $C_{12}(y, 1-y; \theta) = 1-\theta(2y-1)^2$. Suppose that θ is positive. Then, $\Psi = \int_0^1 C_{12}(F(x), 1-F(x)) f(1-x) f(x) dx = \int_0^1 \left[1 - \theta(2F(x) - 1)^2\right] f(1-x) f(x) dx$, and therefore we obtain

$$\frac{d\Psi}{d\theta} = -\int_0^1 (2F(x) - 1)^2 f(1-x) f(x) dx < 0,$$

which implies that π^B increases in θ. The case with a negative θ can be analyzed analogously. ■

4 Some intuitive explanation

In the previous section we utilized copula techniques to derive a sufficient condition for the profitability of interfirm bundling relative to separate sales. Although copulas are useful for describing the dependence structure among random variables, they are not particularly helpful to obtain economic intuitions behind the result. So, in this section, we characterize and compare equilibrium prices and profits using a traditional methodology, so as to find the underlying economic logic making interfirm bundling more profitable than separate sales.

Recall that under interfirm bundling the market boundary is given by the following indifference condition:

$$tx_1 + tx_2 + p_{1A} + p_{2A} = t(1 - x_1) + t(1 - x_2) + p_{1B} + p_{2B}.$$

This straight line, which is orthogonal to the 45-degree line, separates consumers into two groups, the one choosing AA and the other choosing BB. In order to calculate the marginal increase in demand resulting from a small price cut (e.g., $-\partial D_A/\partial p_{iA}$), we first calculate how much the market boundary moves along the 45-degree line when firm iA reduces its price by Δp. For a given x_j, if firm iA reduces its price by Δp, the market boundary moves along x_i axis as much as $\Delta p/2t$. By projecting two-dimensional vector $(\Delta p/2t, 0)$ on the 45-degree line, one can see that, in response to the price change, the indifferent type moves from (x_1, x_2) to $(x_1 + \Delta p/4t, x_2 + \Delta p/4t)$ along the 45-degree line. Then, by the Pythagorean theorem, the distance by which the market boundary moves along the 45-degree line is calculated as

$$\sqrt{2\left(\frac{\Delta p}{4t}\right)^2} = \frac{\Delta p}{2t\sqrt{2}}.$$

Note that this distance is $1/\sqrt{2}$ times the distance the market boundary moves following the same amount of price reduction in the one-dimensional Hotelling model, which is given by $\Delta p/2t$. The market boundary moves less sensitively under interfirm bundling than under separate sales to a price change, which means that demand is less elastic under interfirm bundling relative to separate sales. This is because when products are bundled a consumer who wishes to switch to a different seller of product i has to switch for product j as well, which makes switching more costly for the consumer compared to switching brand for a single product separately. The difference in switching costs between bundling and separate sales is magnified as the number of products increases.

Then, the change of firm ij's demand due to price increase by Δp_{ij} can be calculated as follows:

$$\Delta D_j \approx -\frac{\Delta p_{ij}}{2t} \times \underbrace{\frac{1}{\sqrt{2}}}_{\text{switching-hindering effect}} \times \underbrace{\Lambda}_{\text{market boundary effect}},$$

where Λ is the mass of indifferent consumers at the equilibrium.[8] The demand change is affected by the three elements: the first term is the demand change in the usual one-dimensional Hotelling model, the second term denotes the switching-hindering effect, and the third the market boundary effect. In words, the change of the demand is approximately the area of the rectangle with width $\Delta p_{ij}/(2t\sqrt{2})$ and height Λ (see Figure 1 below). The switching-hindering effect has not been fully recognized in the literature, while the market boundary effect has been received a fair amount of attention in several aspects (see Section 5 for more details). Here we wish to highlight the importance of the switching-hindering effect in evaluating the overall price effect of competitive bundling.

Dividing both sides by Δp_{ij} and taking the limit, we obtain

$$\frac{\partial D_j}{\partial p_{ij}} = \lim_{\Delta p_{ij} \to 0} \frac{\Delta D_j}{\Delta p_{ij}} = -\frac{\Lambda}{2t\sqrt{2}}$$

Imposing symmetry ($D_j(p) = 1/2$) on the first-order condition of firm ij's profit-

[8] We can measure the mass of indifferent consumers as follows. Let us define a function $h(z) \equiv g\left(z/\sqrt{2}, 1 - z/\sqrt{2}\right)$. Recall that g is the joint density function of X_1 and X_2. The domain of $h(\cdot)$ is the straight line $x_2 = 1 - x_1$. Note that when x_1 moves from 0 to 1, z moves from 0 to $\sqrt{2}$, i.e., $dz = \sqrt{2}dx$. So, the mass of indifferent consumers on the line $x_2 = 1 - x_1$ is given by

$$\begin{aligned}\Lambda &\equiv \int_0^{\sqrt{2}} h(z)dz = \int_0^{\sqrt{2}} g\left(\frac{z}{\sqrt{2}}, 1 - \frac{z}{\sqrt{2}}\right) dz \\ &= \sqrt{2}\int_0^1 g(x, 1-x)\, dx.\end{aligned}$$

maximization problem ($p_{ij} = -D_j(p)\frac{\partial p_{ij}}{\partial D_j}$), we can rewrite the equilibrium price of each product and each firm's profit under interfirm bundling as follows:

$$p^B = \frac{t}{\Lambda/\sqrt{2}}, \pi^B = \frac{t}{2\Lambda/\sqrt{2}}. \tag{3}$$

Then the following result is immediate from the comparison of equilibrium prices and profits in (2) and (3).

Proposition 3 *The prices and profits are higher under interfirm bundling compared with separate sales if and only if* $\Lambda/\sqrt{2} < f(1/2)$.

From (2) and (3) it must be that $\Psi = \Lambda/\sqrt{2}$. This means that the marginal demand effect of a price cut under interfirm bundling can be decomposed into two parts: Λ representing the mass of consumers at the market boundary and $1/\sqrt{2}$ representing the rigidity of demand due to additional switching costs under bundling. The price effect of bundling is jointly determined by these two factors, and so is the overall profitability of interfirm bundling.

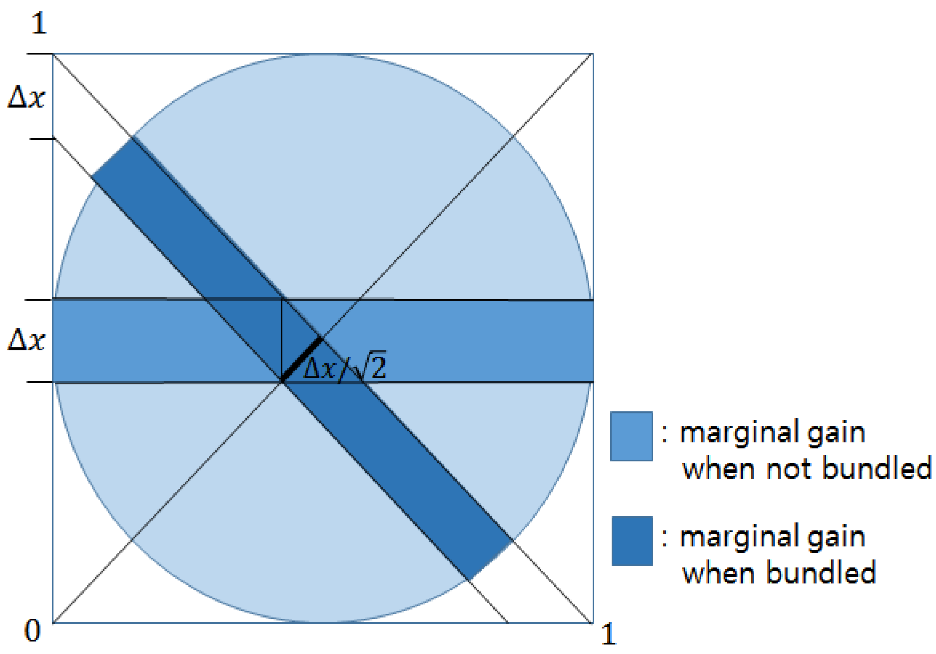

Figure 1. Price effects under interfirm bundling and separate sales

Example: Suppose that $G(x_1, x_2)$ is the uniform distribution on a disk of radius $1/2$. In this case, the density of the equilibrium market boundary is the same under bundling and separate sales as $\Lambda = f(1/2)$. In fact, this property holds for all radial distributions (e.g. a joint normal distribution with zero correlation). Then, the market boundary effect is canceled out and the price effect of bundling is solely affected by the switching-hindering effect, and therefore the price and profit are $\sqrt{2}$ times higher under interfirm bundling relative to separate sales. Figure 1 shows how the marginal increase in demand due to a price cut differs in the two regimes. Note that as the support of the distribution expands toward the four corners of the unit square the density of the market boundary becomes larger under bundling while it remains constant under separate sales. This makes price competition more intense under interfirm bundling compared with the unit disk case. Interestingly, when consumers are uniformly distributed in the unit square this competition-intensifying effect exactly cancels out the switching hindering effect, resulting in the identical price and profits in the two regimes, as shown by Matutes and Regibeau (1988). Then, we can easily figure out that, if the support shrinks from the four corners of the unit square even slightly, the market boundary effect is dominated by the switching-hindering effect, leading to higher prices and profits under interfirm bundling relative to separate sales.

The above analysis can be easily extended to the case of $n \geq 2$ products, each supplied by two horizontally differentiated firms. The indifferent type $(x_1, ..., x_n)$ must satisfy

$$t\left(\sum_{i=1}^{n} x_i\right) + \sum_{i=1}^{n} p_{iA} = t\left(\sum_{i=1}^{n}(1 - x_i)\right) + \sum_{i=1}^{n} p_{iB}.$$

The distance by which the market boundary moves as a result of price change Δp under interfirm bundling is

$$\sqrt{n\left(\frac{\Delta p}{2nt}\right)^2} = \frac{\Delta p}{2t\sqrt{n}},$$

and the derivative of the demand is given by

$$\frac{\partial D_j}{\partial p_{ij}} = \lim_{\Delta p_{ij} \to 0} \frac{\Delta D_j}{\Delta p_{ij}} = -\frac{\Lambda_n}{2t\sqrt{n}},$$

where Λ_n is, as before, the mass of indifferent consumers at the equilibrium. Using the first-order condition, we can obtain the following result.

Proposition 4 *The symmetric equilibrium price and profit under n-product interfirm bundling are*

$$p^B(n) = \frac{t}{\Lambda_n/\sqrt{n}}, \pi^B(n) = \frac{t}{2\Lambda_n/\sqrt{n}},$$

and interfirm building is profitable if and only if $\Lambda_n/\sqrt{n} < f(1/2)$.

5 Related literature

Our analysis has identified two major elements governing the price effect of interfirm bundling in oligopoly. One is the switching-hindering effect which says that bundling makes it more difficult for consumers to switch brand. It always works in a way of reducing the intensity of price competition. The other is the market boundary effect which means that the intensity of competition is affected by the mass of indifferent consumers at the equilibrium. Since the relative density of indifferent consumers under the two regimes depends on the shape of the valuation distribution, it is hard to tell a priori whether price competition is loosened or intensified by the market boundary effect.

The existing literatures on competitive bundling have not successfully distinguished those two effects. Most of previous works focused on the situation where consumers are uniformly distributed on the unit square. In this case, the market boundary effect works against the switching-hindering effect, and interestingly those two are exactly canceled out, nullifying the price effect of bundling. The earliest case is the mix-and-match model proposed by Matutes and Regibeau (1988). They explicitly showed that equilibrium prices and profits are identical under compatibility and incompatibility, each corresponding to separate sales and interfirm bundling respectively. Since consumers are uniformly distributed on the unit square, $f(x) = 1$ for all $x \in [0, 1]$ and therefore the equilibrium profit under compatibility is given by $\pi^S = t/2f(1/2) = t/2$. On the other hand, under incompatibility the mass of indifferent consumers on the diagonal line (i.e., the length of the line under the uniform assumption) is given by $\Lambda = \sqrt{2}$ and so the equilibrium profit is $\pi^B = \sqrt{2}t/(2\Lambda) = t/2$, identical to the profit obtained under separate sales. Based on these calculations, Matutes and Regibeau conclude that firms do not have strict incentives to bundle their products with others selling complementary components. We know from Proposition 1 that this result is an exception rather than the rule: it holds only in the knife-edge case of the uniform distribution on the unit square. Gans and King (2006) reached a similar conclusion in their analysis of interfirm bundled discounts. Using the exactly same two-dimensional Hotelling framework, they show that firms' profits are the same with or without bundled discounts.[9]

Follow-up studies have shown that the profit-neutrality result may not hold if some asymmetries are introduced into the standard Hotelling model. Denicolo (2000) and Hermalin and Katz (2013) found that if the degree of (horizontal) differentiation differs between the products the firms can obtain larger profits under interfirm bundling, even if the assumption of the uniform distribution on the unit square is maintained. This is because with products differing in the degree of differentiation the density of market boundary under interfirm bundling, in some cases, becomes so small relative to the case of separate sales that the switching-hindering effect outweighs the market boundary effect. Specifically, Hermalin and Katz showed that relatively undifferentiated platforms can increase joint profits using bundling arrangements with relatively differentiated ap-

[9] In their model, however, two pairs of firms in fact choose to offer bundled discounts because their profits without discounts are even lower when the rival pair of firms offer bundled discounts, resulting in a sort of prisoners' dilemma situation.

plications. On the other hand, Denicolo showed that the generalist firm selling two components of a system, facing competition from two specialist firms each supplying one component only, may have an incentive to choose incompatibility (bundling).

The relative strength of the market boundary effect between the two regimes is also affected by the degree of cost or quality asymmetry between the firms. Hahn and Kim (2012) have shown that if competing suppliers are asymmetric in production cost or product quality the efficient firms have joint incentives to bundle their products (incompatibility) and all firms including the inefficient benefit from interfirm bundling arrangements. Bundling reduces heterogeneity in consumer valuations, inducing a more concentrated valuation distribution with thinner tails, compared with the valuation distribution of a single product. Cost or quality asymmetry naturally moves the market boundary away from the middle. With interfirm bundling, however, the degree of asymmetry is amplified (because one bundling arrangement is formed between the efficient firms and the other is between the inefficient firms), and therefore the equilibrium market boundary moves closer to either tail. As a result, if the density of the market boundary becomes sufficiently low under interfirm bundling, the price competition becomes less intense with bundling relative to separate sales. Hurkens at al. (2016) derived a qualitatively similar result in the context of dominance and foreclosure. They also extend the intuition to a more general class of symmetric and log-concave densities.

Kim and Choi (2015) considered a torus on which firms are symmetrically located and consumers are uniformly distributed (i.e., two-dimensional circular city). Using this framework they allow for more than two competing firms for an individual product and show that at least one symmetric equilibrium exists in which firms' profits are higher under incompatibility (i.e. bundling) compared with compatibility (separate sales). However, they also fail to distinguish the switching-hindering effect from the market boundary effect.

A recent work by Zhou (2017) is worth mentioning. Interestingly, he departs from spatial models and adopts a random utility approach à la Perloff and Salop (1985) to analyze the price effect of competitive bundling. He shows that bundling by firms offering multiple products raises prices and profits, relative to separate sales, when the number of firms exceeds a threshold. This result is mainly driven by a comparative static analysis of the market boundary effect with respect to the number of firms. The focus of his analysis is on unilateral bundling by multi-product firms, unlike interfirm bundling between independent single-product firms in our model.

All these previous analyses failed to disentangle the market boundary effect and the switching-hindering effect. In fact, most of them treated the switching-hindering effect as a part of the market boundary effect. In an earlier work (Hahn and Kim, 2016), we did recognize the importance of the switching-hindering effect in analyzing the competitive effect of interfirm bundled discounts. However, the market boundary effect was absent in the model since we assumed that one product is homogeneous while the other is differentiated à la Hotelling.[10] In the present paper, we clearly distinguish those two effects and highlight the distinct role of the switching-hindering effect in determining the overall price effect of interfirm bundling. In sum, we showed that interfirm bundling is generally profitable, provided that the distribution of consumer valuations is more or less symmetric so that the market boundary effect becomes neutral under bundling and separate sales.

[10] When the transportation cost in market 1 is t_1 and that in market 2 is t_2, the marginal change of demand is given by

$$\frac{\partial D_j}{\partial p_{ij}} = -\frac{\Lambda}{2\sqrt{t_1^2 + t_2^2}}.$$

Since in Hahn and Kim (2016), market 1 is for a homogeneous good, $\partial D_j / \partial p_{ij} = -\Lambda/2t_2$, so the sellers of the homogeneous good enjoy a strictly positive profit.

6 Conclusion and discussion

Using the copula representation of consumer preferences, we showed that interfirm bundling is more profitable than separate sales as long as the distribution of consumer valuations for the products are sufficiently symmetric and the marginal density is quasi-concave. In such a case, independent firms have mutual incentives to bundle their products or make exclusive dealing arrangements. We highlighted the switching-hindering effect of bundling and its role in relaxing competition, which has been largely neglected in the literature.

We conclude with two final remarks. First, the same logic and intuition can be applied to bundling by multiproduct firms, provided that consumers are distributed in the Hotelling product space. In fact, the switching-hindering effect has been present in most of previous analyses on competitive bundling since Matutes and Regibeau (1988), although it has not been notified explicitly. Second, one may ask what happens if the firms can sell their product separately together with the bundle. Allowing mixed bundling would make the analysis very complicated. Nevertheless, we can easily see that the main results obtained under pure bundling still go through under mixed bundling, provided the consumer values for the products are highly correlated (either positively or negatively) or independent. If the consumer values are highly positively correlated, most consumers are located near the diagonal line and, therefore, it is just like pure bundling. The same reasoning can be applied the case of negative correlation, since we can always relocate the firms on the product space to generate positively correlated consumer values. On the other hand, if the consumer values are independent, we can resort on the result of Gans and King (2006) who showed that in the Hotelling framework with the unit square two pairs of firms offer a sufficiently large bundled discount so that all consumers buy one of two bundles, i.e. pure bundling, in equilibrium.

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