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Abstract

We consider a simple auction setting where there are three bidders and one of the bidders creates positive or negative externalities on the other two bidders. We theoretically and experimentally compare two auction formats, the first-price auction (FPA) and the second-price auction (SPA), in our setting. Using a refinement of undominated Nash equilibria, we analyze equilibrium bids and outcomes in the two auction formats. Our experimental results show that overbidding relative to equilibrium bids is prevalent, especially in the SPA, and this leads to higher revenues and lower efficiency in the SPA than in the FPA, especially under negative externalities. With incomplete information, we observe similar tendencies, while we obtain no evidence for learning effects.

Keywords: auctions; externalities; experiments; overbidding; efficiency.

JEL: C91; D44; D62.

1 Introduction

Auctions are widely used to allocate items or resources, and bidders participating in an auction may care about the winning bidder for various reasons. For example, if a telecom-

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munications company obtains a frequency band in a spectrum auction, it can offer better services to its customers, and its rival companies may suffer from reduced market shares. While negative externalities are natural among competing firms, a winning bidder can also create positive externalities on others. For instance, if a person wins a painting at an auction and displays it in his house, a close friend of his who often visits his house will benefit from his winning. Externalities occurring among auction participants can be positive or negative, and in addition they can be *identity-dependent* in the sense that some bidders (e.g., those having stronger rivalry or friendship) impose or incur greater externalities than others do.

In this paper, we aim to study auction mechanisms in a setting where there are positive or negative identity-dependent externalities among bidders. In particular, we theoretically and experimentally investigate two well-known sealed-bid auction formats, namely, the first-price auction (FPA) and the second-price auction (SPA). Although more complicated auction mechanisms may perform better in the presence of externalities, we focus on these two auction formats because they are widely used in practice and have simple rules that participants in an experiment can easily understand. In addition, we consider the following relatively simple setting in order to facilitate our theoretical and experimental investigation. There are three bidders who participate in an auction of an indivisible object. One of the three bidders is called Red and the other two Blue. The Red bidder's winning the object creates the same level of externalities to the Blue bidders,[1] while a Blue bidder's winning imposes no externalities. Though simple, this setting allows us to capture interesting features of externalities in that we can deal with identity-dependent externalities that are positive or negative.

Our main focus in our theoretical and experimental studies is on the complete information scenario where the three bidders' valuations of the object and the level of externalities exerted by the Red bidder are common knowledge among the bidders.[2] In our theoreti-

[1] That is, we assume that externalities do not depend on the affected party's identity.

[2] The assumption of complete information is especially relevant to situations where bidders participate in the same kinds of auctions (for example, spectrum auctions and procurement auctions) repeatedly so that they get to know each other's types. See, for example, Andreoni et al. (2007), Bae and Kagel (2019), and Che et al. (2017) for experimental studies that consider a complete information scenario.

cal analysis, we study noncooperative equilibria of the games induced by the two auction formats. In order to reduce the multiplicity of equilibria, we propose a refinement of undominated Nash equilibrium called *effectively undominated Nash equilibrium* and adopt it as the equilibrium concept for our analysis. We characterize effectively undominated Nash equilibria under various conditions on the three bidders' valuations and the level of externalities.

In the benchmark case of no externalities, it is well-known that both auction formats allocate the object to the bidder with the highest valuation and achieve the revenue equal to the second highest valuation at any undominated Nash equilibrium. In the presence of externalities, we can consider two cases regarding efficient allocations: one where it is efficient for the bidder with the highest valuation to obtain the object, and the other where the presence of externalities makes it inefficient for the bidder with the highest valuation to obtain the object. The former case occurs if the Red bidder has the highest valuation under positive externalities, or if a Blue bidder has the highest valuation under negative externalities. We show that the results for the case of no externalities can be generalized to the former case. The latter case, which is more interesting in our view, occurs if the Red bidder does not have the highest valuation but it is efficient for her to obtain the object accounting for positive externalities, or if the Red bidder has the highest valuation but it is inefficient for her to obtain the object accounting for negative externalities. We show that the equilibrium allocation can be efficient or inefficient depending on the valuations and the level of externalities in the latter case. We study the two cases for both positive and negative externalities and cover the total four cases in Propositions 1–4.

Based on our theoretical results, we make three predictions for our experiments: (1) both auction formats yield the same allocation and revenue, (2) Blue bidders' bids and the revenue decrease in the level of externalities when the Red bidder wins the object, and (3) inefficient allocations are more likely when there are inefficient equilibria than when there are only efficient equilibria. In order to test these predictions, we conducted laboratory experiments with the two treatments of the FPA and the SPA. In our experiments, we used predetermined parameter sets for the valuations and the level of externalities so that we can cover the cases of positive and negative externalities evenly and focus on the more interesting case where it is inefficient for the bidder with the highest valuation to receive the object.

Our experimental data reveal that participants tend to overbid relative to equilibrium bids, especially in the SPA. This result that participants overbid more in the SPA than in the FPA has been reported in the extant experimental literature (see, for example, Kagel, 1995), and various behavioral motives such as spitefulness and joy of winning have gained attention in explaining observed overbidding behavior (see, for example, Andreoni et al., 2007; Cooper and Fang, 2008; and Kimbrough and Reiss, 2012).[3] We find that the two auction formats achieve similar revenues and efficiency under positive externalities, consistently with Prediction 1, but not under negative externalities, where the SPA yields higher revenue and less efficient allocations than the FPA. This finding suggests that standard models have higher explanatory power for the case of positive externalities. In order to test Prediction 2, we focus on the case of positive externalities because the Red bidder wins at equilibrium only under positive externalities in our experiments. We find that Blue bidders' bids and the revenue decrease in the level of externalities under positive externalities, consistently with Prediction 2, except that the predicted effect of externalities on Blue bidders' bids are not statistically significant in the SPA. Lastly, our experimental data contradict Prediction 3 as the existence of inefficient equilibria is shown to have no significant effect on efficiency.

In our experiments, we also implemented an incomplete information scenario where each bidder knows only her own valuation and the level of externalities. Specifically, we ran the last 10 rounds among the total 25 rounds with incomplete information, keeping the same groups and the same parameters. We included these incomplete information rounds in our experiments in order to study whether our experimental findings with complete information are robust to the lack of information about the other bidders' valuations and whether learning effects influence the outcomes of the auction mechanisms. We obtain qualitatively similar results in the two information regimes, while we find no evidence for learning effects.

[3] Bartling and Netzer (2016) show that cognitive skills are negatively correlated with overbidding, and Filiz-Ozbay and Ozbay (2007) show that the feeling of regret can explain overbidding behavior in the FPA.

1.1 Related Literature

Since the seminal paper by Jehiel et al. (1996), a large theoretical literature has developed on the topic related to auction mechanisms in the presence of externalities. Jehiel et al. (1996) construct an optimal auction with externalities in complete and private information scenarios. Jehiel et al. (1999) study a multidimensional mechanism design problem in another private information scenario.[4] Das Varma (2002) examines equilibrium bidding behavior in the open ascending-bid auction with identity-dependent externalities, while Aseff and Chade (2008) solve an optimal multi-unit auction design problem with identity-dependent externalities. Brocas (2013) and Belloni et al. (2017) investigate mechanism design with negative externalities, whereas Gravin and Lu (2013) study digital goods auctions with positive externalities. Recently, Jeong (2019) proposes multidimensional second-price and English auctions with externalities and studies their properties, while Jeong (2020) analyzes the core of a cooperative auction game with externalities.

Compared to the large theoretical literature on auctions with externalities, the experimental literature on this topic is surprisingly small. In particular, to the best of our knowledge, there are only a few papers investigating relative performances of various auction mechanisms in terms of important measures such as revenue and efficiency. Hu et al. (2013) study a situation in which an entrant's winning imposes negative externalities on two incumbents, and they compare an English ascending price auction and a first-price sealed-bid auction in terms of bidding behavior, revenue, and efficiency. Goeree et al. (2013) consider a multi-unit auction environment and compare the performance between an ascending auction and a discriminatory auction, focusing on the incentive for demand reduction and preemptive bidding. Remotely related, Bagchi and Shor (2006) experimentally study a situation in which one or two licenses are auctioned, and they find that participants underbid relative to their theoretical benchmarks for auctions of one license but overbid when two licenses are auctioned. In relation to these papers, we allow both positive and negative externalities in our experiments, and we compare two well-known sealed-bid auction mechanisms, the

[4] See also Jehiel and Moldovanu (1995, 1996, 2000) for studies on related models and Caillaud and Jehiel (1998) for a study on collusion in auctions with externalities.

first-price and second-price auctions.

The rest of the paper is organized as follows. Section 2 presents theoretical analysis of our setting for the two auction formats. Section 3 describes our experimental design and procedures, and Section 4 provides theoretical predictions for our experiments. Section 5 shows our main experimental results, and Section 6 concludes. Proofs of the propositions are presented in Appendix A, and the experimental instructions in Appendix B.

2 Theoretical Analysis

There are three bidders (called bidders 1, 2, and 3) and an indivisible object. When bidder i receives the object, she obtains utility $v_i \geq 0$, for all $i = 1, 2, 3$. In addition, when bidder 1 receives the object, each of bidders 2 and 3 obtains utility $e \in \mathbb{R}$. That is, bidder 1's obtaining the object creates externalities on the other bidders, while bidder $j \neq 1$ exerts no externalities on the others. In this sense, we consider externalities that depend on the identity of the imposer. We allow both positive and negative externalities, and thus there is no restriction on the sign of e. We refer to v_i as bidder i's *valuation* of the object, e as the *level of externalities*, and $|e|$ as the *size of (positive/negative) externalities*. We assume that the valuations are distinct across the bidders. In our theoretical analysis, we focus on a complete information scenario in which the valuations and the level of externalities are commonly known among the bidders.

We consider two auction formats to allocate the object, the first-price auction (FPA) and the second-price auction (SPA). In each auction format, each bidder i simultaneously submits a bid $b_i \geq 0$, and the bidder who submits the highest bid wins the object. The winning bidder pays the highest bid in the FPA and the second highest bid in the SPA. In the following, we study equilibria of the games induced by the FPA and the SPA, considering the three cases of no, positive, and negative externalities.

2.1 No Externalities

As a benchmark, we first consider the case where there are no externalities (i.e., $e = 0$). In this case, all the three bidders are symmetric in the externality structure, and we assume that $v_1 > v_2 > v_3$ without loss of generality.

Let us consider the game induced by the FPA. It can be shown that a bid profile (b_1, b_2, b_3) is a Nash equilibrium if and only if $b_1 \in [v_2, v_1]$, $b_1 \geq b_j$ for all $j \neq 1$, and $b_1 = b_j$ for some $j \neq 1$, assuming that ties are broken in favor of a bidder with a lower index.[5] At any Nash equilibrium, bidder 1 obtains the object, and thus the efficient allocation of the object is achieved.[6] Since bidding more than one's valuation is weakly dominated in the FPA, a bid profile (b_1, b_2, b_3) is an undominated Nash equilibrium if and only if $b_1 = b_2 = v_2$ and $b_3 \leq v_3$. Hence, bidder 1 pays the second highest valuation, v_2, at any undominated Nash equilibrium.

Let us consider the game induced by the SPA. Every bidder has a weakly dominant strategy of bidding one's own valuation in the SPA. Hence, the bid profile $(b_1, b_2, b_3) = (v_1, v_2, v_3)$ is the unique undominated Nash equilibrium. At the undominated Nash equilibrium, bidder 1 obtains the object and pays v_2. In addition, there are Nash equilibria where bidders use weakly dominated strategies. For example, a bid profile (b_1, b_2, b_3) such that $b_i > v_1$ and $b_j = 0$ for all $j \neq i$ is a Nash equilibrium for all $i = 1, 2, 3$, and an inefficient allocation can arise at such a Nash equilibrium.

In summary, in both auction formats, bidder 1 obtains the object and pays v_2 at any undominated Nash equilibrium. That is, if we focus on undominated Nash equilibria, both auction formats yield the efficient allocation and the revenue equal to the second highest valuation.

[5] Consider an alternative scenario where the bid space is discrete and ties are broken with equal probability, which is the case in our experiments. Let $\Delta > 0$ be the unit of bids, and let us assume that the valuations are multiples of Δ. In this scenario, a bid profile (b_1, b_2, b_3) such that $b_1 \in [v_2, v_1 - \Delta]$, $b_1 > b_j$ for all $j \neq 1$, and $b_1 = b_j + \Delta$ for some $j \neq 1$ is a Nash equilibrium. As Δ goes to zero, any such Nash equilibrium (b_1, b_2, b_3) approaches one with $b_1 = b_j$ and the tie broken in favor of bidder 1. With this interpretation in mind, when we look for Nash equilibria where a particular bidder obtains the object, we will break ties in favor of that bidder.

[6] In our analysis, we assume that bidders have quasilinear utility functions, and thus an efficient allocation of the object maximizes the sum of bidders' utilities including those from externalities.

2.2 Positive Externalities

We next consider the case where bidder 1 creates positive externalities on bidders 2 and 3 (i.e., $e > 0$). Since bidders 2 and 3 are symmetric in the externality structure, we assume that $v_2 > v_3$ without loss of generality. We say that bidder j competes with bidder $k \neq j$ at the bid profile (b_1, b_2, b_3) if bidder k is the highest bidder among the bidders other than bidder j. That is, when bidder j competes with bidder k, bidder j needs to outbid bidder k in order to become the highest bidder. For all $j \neq 1$, let $\tilde{v}_j = v_j - e$, and we call $\tilde{v}_j$ bidder j's *effective valuation* against bidder 1. For any $j \neq 1$, bidder j's maximum willingness to pay for the object is given by her effective valuation if she competes with bidder 1, while it is given by her valuation if she competes with bidder $k \neq 1, j$. When there are positive externalities, bidder j's effective valuation is lower than her valuation (i.e., $\tilde{v}_j < v_j$) for all $j \neq 1$. In the case of positive externalities, we refine undominated Nash equilibria as follows. First, for both auction formats, we require that $b_j \leq \tilde{v}_j$ if bidder $j \neq 1$ competes with bidder 1. Second, for the SPA, we also require that $b_j = v_j$ if bidder $j \neq 1$ does not compete with bidder 1. We refer to an undominated Nash equilibrium satisfying these two requirements as an *effectively undominated Nash equilibrium*. These requirements can be interpreted as eliminating bids of bidder $j \neq 1$ that cannot be justified given her correct belief about whether she competes with bidder 1 or not. In subsequent theoretical analysis, we take an effectively undominated Nash equilibrium as our equilibrium concept, and we sometimes simply refer to it as an equilibrium.

We use different valuations and levels of externalities in our experiments (as listed in Table 1 in Section 3), and we can classify those used in the case of positive externalities into two cases.

Case P1. $v_1 > v_2$

In this case, bidder 1 has the highest valuation, and it is efficient for bidder 1 to obtain the object. In the following proposition, we study the allocation and the revenue at equilibrium in the two auction formats in this case.

Proposition 1. *Suppose that $v_1 > v_2 > v_3$ and $e > 0$. In both auction formats, bidder 1 obtains the* *object and pays $\tilde{v}_2$ at any effectively undominated Nash equilibrium.*

All the proofs of the propositions in this section are presented in Appendix A, and they describe equilibrium bid profiles. For the SPA, there are inefficient equilibria, as in the case of no externalities. However, if we focus on effectively undominated Nash equilibria, both auction formats achieve the efficient allocation and the revenue equal to the higher of the non-winning bidders' effective valuations. As the size of positive externalities becomes larger, bidders 2 and 3 bid less aggressively against bidder 1, and thus the revenue decreases. As the size of positive externalities approaches zero, equilibrium bids converge to those in the case of no externalities.

Case P2. $v_1 + 2e > v_2 > v_1$

In this case, bidder 1 does not have the highest valuation, but the size of positive externalities is large enough to make it efficient for bidder 1 to obtain the object.

Proposition 2. *Suppose that $v_1 + 2e > v_2 > v_1$, $v_2 > v_3$, and $e > 0$. In both auction formats, the following statements hold.*

(i) There is an effectively undominated Nash equilibrium where bidder 1 obtains the object if and only if $v_1 \geq \tilde{v}_2$, and she pays $\tilde{v}_2$ at any such equilibrium.

(ii) There is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $\tilde{v}_2 \geq v_1 > v_3$ or $v_3 > v_1$, and she pays $\max\{v_1, v_3\}$ at any such equilibrium.

(iii) There is no effectively undominated Nash equilibrium where bidder 3 obtains the object.

When the size of positive externalities is large enough to have $v_1 \geq \tilde{v}_2$, the efficient allocation is achieved at equilibrium in both auction formats. On the contrary, when the size of positive externalities is not so large or bidder 1 has the lowest valuation, an inefficient allocation can arise at equilibrium. If $v_3 > v_1 \geq \tilde{v}_2$, there are both efficient and inefficient equilibria. In this case, competition between bidders 2 and 3 may result in the inefficient equilibrium where bidder 2 obtains the object, but they prefer enjoying positive externalities from bidder 1's winning at the efficient equilibrium.[7]

[7] Note that $v_2 - \max\{v_1, v_3\} \leq v_2 - v_1 < 2e$ in Case P2. Thus, the total payoff of bidders 2 and 3 is higher when bidder 1 receives the object than when bidder 2 does at the price $\max\{v_1, v_3\}$.

2.3 Negative Externalities

Lastly, we consider the case where bidder 1 creates negative externalities on bidders 2 and 3 (i.e., $e < 0$). Again, we assume that $v_2 > v_3$ without loss of generality. When there are negative externalities, bidder j's effective valuation against bidder 1 is higher than her valuation (i.e., $\tilde{v}_j > v_j$) for all $j \neq 1$. Hence, in the concept of effectively undominated Nash equilibria for the case of negative externalities, we require that $b_j \leq v_j$ if bidder $j \neq 1$ does not compete with bidder 1, and for the SPA, we also require that $b_j = \tilde{v}_j$ if bidder $j \neq 1$ competes with bidder 1. We classify our experimental settings with negative externalities into two cases.

Case N1. $v_2 > v_1$

In this case, bidder 2 has the highest valuation, and it is efficient for bidder 2 to obtain the object.

Proposition 3. *Suppose that $v_2 > v_1$, $v_2 > v_3$, and $e < 0$. In both auction formats, bidder 2 obtains the object and pays* $\max\{v_1, v_3\}$ *at any effectively undominated Nash equilibrium.*

When there are no externalities, the bidder with the highest valuation wins the object at any undominated Nash equilibrium. The presence of negative externalities increases bidder 2's maximum willingness to pay for the object when she competes with bidder 1, while it makes no difference when she competes with bidder 3. As a result, the efficient allocation is achieved at equilibrium even when there are negative externalities.

Case N2. $v_1 > v_2 > v_1 + 2e$

In this case, bidder 1 has the highest valuation, but the size of negative externalities is large enough to make it not efficient for bidder 1 to obtain the object.

Proposition 4. *Suppose that $v_1 > v_2 > v_1 + 2e$, $v_2 > v_3$, and $e < 0$. In both auction formats, the following statements hold.*

(i) There is an effectively undominated Nash equilibrium where bidder 1 obtains the object if and only if $v_1 \geq \tilde{v}_2$, and she pays $\tilde{v}_2$ at any such equilibrium.

(ii) There is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $\tilde{v}_2 \geq v_1$, and she pays v_1 at any such equilibrium.

(iii) There is an effectively undominated Nash equilibrium where bidder 3 obtains the object if and only if $\tilde{v}_3 \geq v_1$, and she pays v_1 at any such equilibrium.

When the size of negative externalities is not so large that $v_1 \geq \tilde{v}_2$ holds, bidder 1, who has the highest valuation, obtains the object at equilibrium, as in the case of no externalities. Since $v_2 - v_1 > 2e$ and bidder 1's equilibrium bid does not exceed her valuation in both auction formats, bidders 2 and 3 can improve their total payoff by having bidder 2 outbid bidder 1. However, when $v_1 \geq \tilde{v}_2$, bidder 2 cannot gain by becoming the highest bidder, unless she receives a compensation from bidder 3. The inability of bidders 2 and 3 to behave collectively in our noncooperative equilibrium concept results in an inefficient allocation in this case.[8] When the size of negative externalities is large enough to have $\tilde{v}_j \geq v_1$ for some bidder $j \neq 1$, bidder j is willing to pay more than her valuation in order to avoid negative externalities resulting from bidder 1's winning the object. When $\tilde{v}_2 \geq v_1 > \tilde{v}_3$, only bidder 2 is willing to be the highest bidder. On the other hand, when $\tilde{v}_3 \geq v_1$, both bidders 2 and 3 can become the highest bidder at equilibrium. If bidder $j \neq 1$ is the highest bidder at equilibrium, her payoff is $v_j - v_1 < 0$ while the other bidder's payoff is zero. If both bidders 2 and 3 choose low bids in the hope that the other bidder becomes the winner, bidder 1 may become the winner.

3 Experimental Design and Procedures

We ran experimental sessions at the Center for Research in Experimental and Theoretical Economics (CREATE) managed by the School of Economics at Yonsei University in South Korea. We experimentally implemented two auction mechanisms of the FPA and the SPA in our study, and we implemented the two treatments with a between-subject design.

In each round, participants were randomly matched into groups of three and given 170 experimental coins each. They were told that they participate in an auction for an item with their group members and that the winner obtains v coins, where v represents each

[8] Jeong (2020) studies the core of an auction game with externalities and transferable utility, where side payments between bidders are possible.

bidder's valuation of the item (corresponding to v_i in Section 2) and can be different across members. In each group, one member is called "Red" and the other two "Blue." Between the two Blue participants, we call the one who has the higher valuation "Blue-High" and the other "Blue-Low." The Red participant's winning the item creates either positive or negative externalities on the Blue participants: if the Red participant wins, she obtains v coins and the Blue participants obtain e coins, where e can be positive or negative. Note that a Red participant corresponds to bidder 1, a Blue-High participant to bidder 2, and a Blue-Low participant to bidder 3 in Section 2.

The rules for bidding and payments were different between the two treatments. In both treatments, each participant made a bid within her budget. That is, a bid was made as an integer between 0 and 170 coins. In the FPA, the highest bidder won the auction and paid her bid. In the SPA, the highest bidder won the auction and paid the second highest bid. Participants were informed that if there are multiple highest bidders, one bidder is randomly chosen among them by the server computer with equal chances. These two auction mechanisms are well known to economists as well as to laypeople.

Participants played this auction game for 25 rounds with feedback about the winner and the bids of the three participants in their groups at the end of each round. The values of valuations v and externalities e were predetermined for ease of comparison. Note that the parameter space for each group is vast, consisting of four parameters one of which can be positive or negative. If we had chosen parameters randomly for each group, the realized parameter sets might have covered different parameter ranges (as classified in Section 2) unevenly between the two treatments, which would have impeded comparison between the treatments. Alternatively, we could have used the same randomly generated parameter set for all the groups in each round. In this case, with less samples, it might have happened that most of the realized parameter sets cover uninteresting cases (e.g., very small sizes of externalities). Given the budget and time constraints, we thus chose to use predetermined parameters in order to enhance comparison between the treatments and focus on more interesting cases.

Table 1: Parameters for the Valuations and the Levels of Externalities

	Round	v Red	Blue-High	Blue-Low	e
Practice	1	72	91	58	−36
	2	56	40	37	−57
	3	64	69	59	0
	4	34	52	37	25
	5	95	88	78	4
CI	6	78	95	85	35
	7	98	94	90	−64
	8	33	70	51	42
	9	94	79	67	−18
	10	81	72	63	−42
	11	42	68	41	52
	12	93	70	59	−28
	13	96	91	90	2
	14	56	63	53	9
	15	79	37	35	−48
II	Positive	46	65	63	38
	Negative	71	64	58	−29

Note: CI = complete information; II = incomplete information.

In Table 1, we list the predetermined parameters, the valuations v for the three participants in each group and the levels of externalities e, that were used in our experiments. In the first 15 rounds, we adopted a random matching protocol, forming groups in each round, and participants played an auction game with all relevant information, i.e., they knew every member's valuation and the level of externalities. Among the 15 rounds, the first 5 rounds were practice rounds, and the next 10 rounds are called the *complete information (CI)* rounds. For instance, consider the parameters for Round 8 in Table 1. In Round 8, the Red participant's valuation is 33, and the two Blue participants' valuations are 70 and 51. The level of externalities is 42, and therefore there are positive externalities created by the Red's winning.

After Round 15, new groups of three participants were randomly formed, and participants played auction games with the same members (i.e., a fixed matching protocol was used) for the final 10 rounds. In these last 10 rounds, participants played auction games

Table 2: Information about Treatments

Treatment	FPA	SPA
Sessions	3	3
# of participants	42	48
Average payments	21,468	20,817

Note: Payments are expressed in KRW.

with a limited amount of information: each participant knew her own valuation and the level of externalities but not the others' valuations. We call these rounds the *incomplete information (II)* rounds. In these rounds, participants may infer the other members' valuations from feedback about their bids. Consider the last row in Table 1. If a group is assigned this set of parameters for Rounds 16–25, the Red participant's valuation is 71, and the two Blue participants' valuations are 64 and 58. The level of externalities is −29, and thus there are negative externalities. In this case, for example, the Red participant knows that her valuation is 71 and the level of externalities is −29, but she does not know the two Blue participants' valuations. Groups of participants were randomly assigned to the two cases of positive and negative externalities. By implementing the II rounds in our experiments, we seek to understand the effects of private information and to check whether our findings in the CI rounds are robust to the lack of information.

Table 2 shows information about sessions. We ran three sessions for each of the FPA and SPA treatments in September 2019. One of the authors led all the sessions to minimize confounding factors. In total, we invited 42 and 48 undergraduate students to the FPA and SPA treatments, respectively, from our subject pool. The experimental instructions for the two treatments are presented in Appendix B.[9] After Round 25, the experiments ended with demographic surveys (i.e., age, gender, major, religion), and one round from Rounds 6–25 was randomly chosen by the server computer for payments to participants. Each coin in the chosen round was converted to KRW 95, and participants obtained gift certificates worth their payoffs. The average payment including show-up fees (KRW 5,000) was about

[9]Because the instructions for the two treatments are identical except for one paragraph, we present them together in Appendix B.

Table 3: Classification of the Parameter Sets and the Summary of Equilibrium Outcomes

Case	Prop.	Bids in FPA			Bids in SPA			Win.	Pay.	Eff.	Rounds
		R	BH	BL	R	BH	BL				
P1	1	$\tilde{v}_2$	$\tilde{v}_2$	$\tilde{v}_3$	v_1	$\tilde{v}_2$	$\tilde{v}_3$	R	$\tilde{v}_2$	eff	13
P2-1	2(i)	$\tilde{v}_2$	$\tilde{v}_2$	$\tilde{v}_3$	v_1	$\tilde{v}_2$	$\tilde{v}_3$	R	$\tilde{v}_2$	eff	6, 8, 11, 14
P2-2	2(ii)	v_1	v_3	v_3	v_1	v_2	v_3	BH	v_3	ineff	6, 8
N2-1	4(ii)	v_1	v_1	v_3	v_1	$\tilde{v}_2$	v_3	BH	v_1	eff	7, 9, 10, 12, 15
N2-2	4(iii)	v_1	v_2	v_1	v_1	v_2	$\tilde{v}_3$	BL	v_1	ineff	7, 10, 15

Note: R = Red bidder, BH = Blue-High bidder, BL = Blue-Low bidder, Win. = Winner, Pay. = Winner's Payment (or Revenue), Eff. = Efficiency, eff = efficient allocation, ineff = inefficient allocation.

KRW 21,000 (around USD 18). Each session for the FPA and SPA treatments took about 50 minutes.

4 Theoretical Predictions

In Table 3, we classify the parameter sets used in the CI rounds in our experiments according to the structures of equilibria. Table 3 presents the cases to which the parameter sets belong, the propositions in which these cases are studied in Section 2, and the bids of the Red bidder (R; bidder 1), the Blue-High bidder (BH; bidder 2), and the Blue-Low bidder (BL; bidder 3) in the FPA and the SPA at effectively undominated Nash equilibria. It also shows the winner and her payment (or the revenue) at equilibrium as well as the efficiency of the equilibrium allocation. Equilibrium bid profiles are derived in the proof of the propositions presented in Appendix A. For the FPA, there is a range of equilibrium bids for the lowest bidder, and we take the upper limit of the range in Table 3. The equilibrium bids shown in Table 3 are derived with the assumptions of a continuous bid set and arbitrary tie-breaking, and we take these values for convenience. If we assume a discrete integer bid set and random tie-breaking as in our experiments, the second highest equilibrium bid in the FPA is reduced by 1, while there is no change in the SPA.

Among the CI rounds, Rounds 6–15, there are positive externalities in Rounds 6, 8, 11, 13, and 14, while there are negative externalities in Rounds 7, 9, 10, 12, and 15. Among the rounds with positive externalities, Round 13 belongs to Case P1 in Section 2, which is studied in Proposition 1. All the other rounds with positive externalities belong to Case P2, which is divided into P2-1 and P2-2 in Table 3. In Case P2-1, which is studied in Proposition 2(i), there are efficient equilibria. On the other hand, in Case P2-2, which has $v_3 > v_1$ and is studied in Proposition 2(ii), there are inefficient equilibria. It is possible that a parameter set belongs to the two cases simultaneously, which is the case for Rounds 6 and 8. All the rounds with negative externalities belong to Case N2, which is divided into N2-1 and N2-2 in Table 3. Cases N2-1 and N2-2 are studied in Proposition 4(ii) and (iii), respectively. There are efficient equilibria in Case N2-1, while there are inefficient equilibria in Case N2-2. The condition for Case N2-2 implies that for Case N2-1, and thus Case N2-2 is a subcase of Case N2-1. Rounds 7, 10, and 15 belong to Case N2-2. From Table 3, it can be seen that we have focused on parameter sets with which we can compare cases where there are only efficient equilibria and those where there are both efficient and inefficient equilibria.

Based on the results in Propositions 1–4 and the classification in Table 3, we can make the following theoretical predictions for our experiments.

Prediction 1. (Comparison between the FPA and the SPA) There is no difference between the FPA and the SPA in terms of the allocation and the revenue.

Prediction 2. (Effects of externalities on bids and revenue) The Blue bidders' bids and the revenue decrease in the level of externalities when the Red bidder wins the object.

Prediction 3. (Occurrence of inefficient allocations) Inefficient allocations are more likely to occur when there are inefficient equilibria than when there are only efficient equilibria.

In all the cases covered in Propositions 1–4, the bidder who obtains the object and her payment are the same in the two auction formats, as long as we focus on effectively undominated Nash equilibria. Thus, we can predict that both auction formats yield the same allocation and revenue. In the cases where the Red bidder obtains the object at equilibrium (covered in Propositions 1, 2(i), and 4(i)), the equilibrium bid of a Blue bidder is given by her effective valuation and the equilibrium revenue is given by the higher of the Blue bidders' effective valuations. Since the effective valuations decrease in the level of externalities, e, we can expect that the Blue bidders' bids and the revenue decrease in e as well when the Red bidder wins the object. In Case P2, if $v_1 \geq \tilde{v}_2$, there is an efficient equilibrium where the Red bidder obtains the object, and if $v_3 > v_1$, there is an inefficient equilibrium as well. Hence, given that $v_1 \geq \tilde{v}_2$ holds, we can predict that inefficient allocations are more likely to occur when $v_3 > v_1$ (Rounds 6 and 8) than when $v_1 > v_3$ (Rounds 11 and 14). In Case N2, if $\tilde{v}_2 \geq v_1$, there are only efficient equilibria where the Blue-High bidder wins the object, and if $\tilde{v}_3 \geq v_1$, there are inefficient equilibria as well where the Blue-Low bidder wins the object. Hence, given that $\tilde{v}_2 \geq v_1$ holds, we can expect that inefficient allocations where the Blue-Low bidder receives the object are more likely to occur when $\tilde{v}_3 \geq v_1$ (Rounds 7, 10 and 15) than when $v_1 > \tilde{v}_3$ (Rounds 9 and 12).

5 Experimental Results

5.1 Complete Information

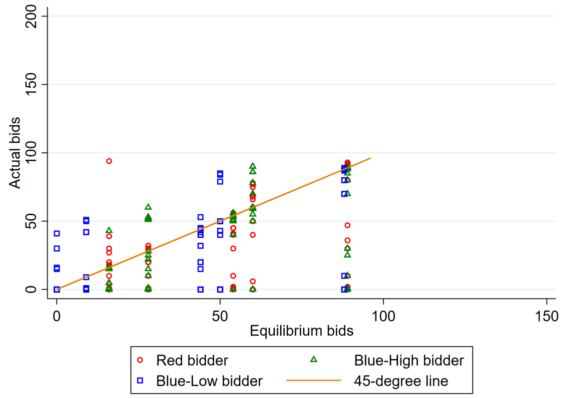
In this subsection, we analyze our experimental data from the complete information rounds in terms of bidding behavior, revenue, and efficiency, testing the theoretical predictions about them.

5.1.1 Bidding Behavior

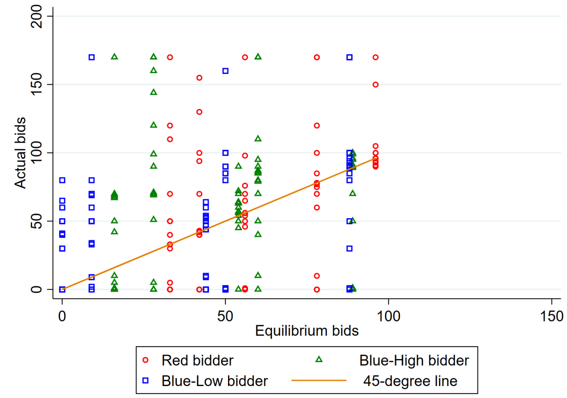
Figure 1 shows bid scatter diagrams where individual (efficient)[10] equilibrium bids are displayed on the horizontal axis and individual actual bids are displayed on the vertical axis. We present four bid scatter diagrams, dividing our bid data depending on the auction format and the sign of externalities. In each plot, different colors and shapes are used to distinguish bidders' roles, and the straight line represents the 45-degree line. These bid scatter diagrams are different from the standard one in that we display equilibrium bids instead of valuations on the horizontal axis. This is because the presence of externalities creates asymmetry among the participants in our experiments. With this difference, the 45-degree line makes it easy to discern whether a bidder overbid relative to her equilibrium bid.

[10] That is, we take efficient equilibria when there are both efficient and inefficient equilibria.

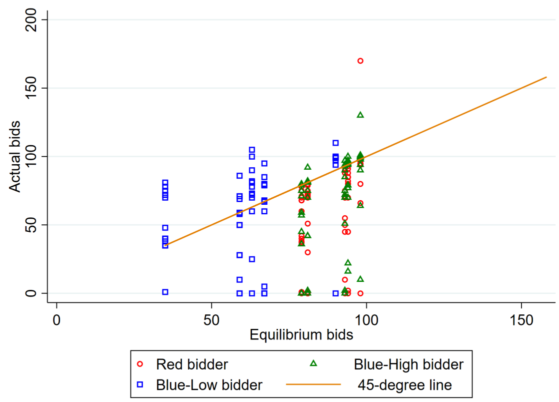
Figure 1: Bid Scatter Diagrams



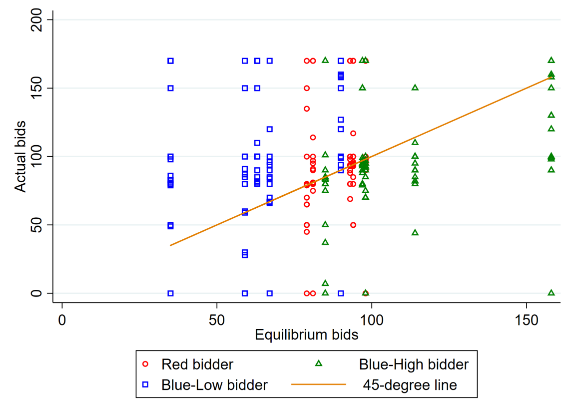
(a) FPA under Positive Externalities



(b) SPA under Positive Externalities



(c) FPA under Negative Externalities



(d) SPA under Negative Externalities

The bid scatter diagrams suggest that participants bid more aggressively in the SPA than in the FPA regardless of the sign of externalities. This finding is consistent with the extant literature showing that overbidding is more widespread in the SPA than in the FPA. We can see in Figure 1 that in the SPA there are many participants who bid the maximum 170 coins or an amount close to it, regardless of their roles. Since the winner does not pay her own bid in the SPA, there are participants who try to win by bidding high and hope that the other members bid low. This kind of behavior is more consistent with Nash equilibria where players use weakly dominated strategies in the SPA than undominated Nash equilibria.

In contrast, the winner pays her own bid in the FPA, and thus participants are more cautious about bidding in the FPA than in the SPA, resulting in less overbidding in the FPA.

However, we can still observe some overbidding in the FPA, notably by Blue-Low bidders under negative externalities, which can be explained as follows. In our concept of effectively undominated Nash equilibria, we assume that each Blue bidder has a correct belief about the bidder with whom she competes and chooses an undominated strategy given the belief. In experiments, however, participants may have uncertainty about the opponents with whom they compete. For example, in the FPA under negative externalities, the Blue-Low bidder is supposed to bid no more than her valuation at efficient equilibria, correctly believing that she competes with the Blue-High bidder. If the Blue-Low bidder mistakenly believes that she competes with the Red bidder, she is willing to bid up to her effective valuation, which is higher than her valuation.

Formally, we report Probit regression results in Table 4, where the dependant variable is the incidence of overbids. The variable takes 1 if the bidder overbids relative to her equilibrium bid and 0 otherwise. The explanatory variables are "SPA," v, e, and "Red," where "SPA" is the indicator variable for the SPA treatment, v is the bidder's own valuation, e is the level of externalities, and "Red" is the indicator variable that represents whether the bidder is a Red bidder or not (i.e., 1 if the bidder is Red and 0 otherwise). Columns (1) and (4) show that in the case of positive externalities, bidders overbid 24.8% points more frequently in the SPA than in the FPA, while in the case of negative externalities, they overbid 16.1% points more often. We find that overbidding is more frequent in the SPA, and the effect is stronger under positive externalities. This confirms our findings from the bid scatter diagrams in Figure 1. In particular, active overbidding by Blue-Low bidders in the FPA under negative externalities results in a weaker effect of the SPA treatment under negative externalities than under positive externalities. Columns (2) and (5) show that bidders tend to overbid more often as the size of externalities increases. As can be seen in Table 3, in the case of positive externalities, a Blue bidder's (efficient) equilibrium bid is her effective valuation. As the size of positive externalities increases, a Blue bidder's equilibrium bid decreases, and it makes overbidding occur more frequently. In the case of negative externalities, overbidding occurs when a Blue bidder uses her effective valuation to determine her bid. Since a Blue bidder's

Table 4: Estimation Results for the Incidence of Overbids

Variables	Incidence of Overbids					
	Positive Externalities			Negative Externalities		
	(1)	(2)	(3)	(4)	(5)	(6)
SPA	0.657***	0.665***	0.664***	0.424***	0.444***	0.478***
	(0.145)	(0.147)	(0.147)	(0.141)	(0.139)	(0.137)
v		0.003	0.003		-0.012***	-0.003
		(0.003)	(0.003)		(0.004)	(0.004)
e		0.008**	0.008**		-0.010***	-0.009***
		(0.003)	(0.003)		(0.003)	(0.003)
Red			-0.016			-0.669***
			(0.168)			(0.182)
Observations	450	450	450	450	450	450
Log-pseudo likl.	-289.8	-287.4	-287.4	-295.9	-286.9	-278.2

Note: Standard errors are clustered at the subject level. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

Table 5: Estimation Results for the Sizes of Overbids

Variables	Overbid Sizes					
	Positive Externalities			Negative Externalities		
	(1)	(2)	(3)	(4)	(5)	(6)
SPA	39.23***	39.25***	39.02***	30.94***	31.55***	33.37***
	(6.878)	(6.777)	(6.957)	(7.290)	(6.956)	(6.970)
v		0.226*	0.206*		-0.668***	-0.410***
		(0.123)	(0.115)		(0.136)	(0.148)
e		0.774***	0.763***		-0.435***	-0.390***
		(0.132)	(0.129)		(0.135)	(0.131)
Red			-2.648			-22.05***
			(7.078)			(6.742)
Observations	450	450	450	450	450	450
Log-pseudo likl.	-1,119.1	-1,106.3	-1,106.2	-1,089.1	-1,073.7	-1,068.2

Note: Standard errors are clustered at the subject level. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

effective valuation increases in the size of negative externalities, overbidding becomes more likely as the size of negative externalities increases. Columns (3) and (6) show that in the case of positive externalities, Red and Blue bidders overbid at a similar rate, while in the case of negative externalities, Red bidders overbid about 24% points less frequently. This is consistent with our observation that Blue-Low bidders tend to overbid a lot, especially in the FPA under negative externalities.

Table 5 provides Tobit regression results where the dependent variable is overbid sizes.[11] Columns (1) and (4) show that, when we consider overbidding bidders, they bid about 30 to 40 more coins (about 18% to 24% of their budgets) in the SPA than in the FPA. Columns (2) and (5) show that bidders overbid more as the size of externalities increases. Columns (3) and (6) show that, in the case of positive externalities, Red and Blue bidders overbid at a similar size, while in the case of negative externalities, Red bidders bid 22 less coins (about 14% of their budgets) than Blue bidders. These results imply that each explanatory variable has a qualitatively similar effect on the incidence of overbids and overbid sizes.

We can summarize our findings on overbidding as follows.

Result 1. *Overbidding is prevalent in both treatments, especially in the SPA and by Blue bidders under negative externalities.*

Because our theoretical predictions in Section 4 are made based on equilibrium analysis, we can expect that there will be more inconsistency with the predictions in situations where overbidding is more severe.

Prediction 2 in Section 4 predicts that the Blue bidders' bids decrease in the level of externalities when the Red bidder wins the object. As can be seen in Table 3, in our experiments, the Red bidder wins only in the case of positive externalities. Table 6 shows that the coefficients for e are all negative but they are economically and statistically significant only in the FPA in columns (1) and (3). Therefore, our data are consistent with the prediction about the Blue bidders' bids in Prediction 2 only in the FPA, not in the SPA. This result is due to the

[11] An overbid size is defined as the difference between the bid and the equilibrium bid if the bid is higher than the equilibrium bid and zero otherwise.

Table 6: Estimation Results for Blue Bidders' Bids under Positive Externalities

Variables	Bids			
	Blue-High Bidders		Blue-Low Bidders	
	FPA	SPA	FPA	SPA
	(1)	(2)	(3)	(4)
v	0.776***	0.675*	0.580***	0.849***
	(0.194)	(0.334)	(0.169)	(0.274)
e	-0.588***	-0.052	-0.348*	-0.050
	(0.152)	(0.258)	(0.194)	(0.275)
Observations	70	80	70	80
R-squared	0.293	0.038	0.268	0.152

Note: Standard errors are clustered at the subject level. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

fact that bidders tend to bid very aggressively in the SPA regardless of the levels of externalities, as we have seen in Figure 1. Note that the absolute value of the coefficient for e in the FPA is smaller for Blue-Low bidders than for Blue-High bidders. This can be explained by Blue-Low bidders' tendency to use their valuations instead of their effective valuations, which results in their overbidding in the FPA. Our results about the effect of externalities on Blue bidders' bids can be summarized as follows.

Result 2. *Under positive externalities, Blue (especially, Blue-High) bidders' bids decrease in the level of externalities in the FPA, not in the SPA.*

To sum up, Blue bidders' bidding behavior is closer to the equilibrium prediction in the FPA than in the SPA, because overbidding is more severe in the SPA.

5.1.2 Revenue

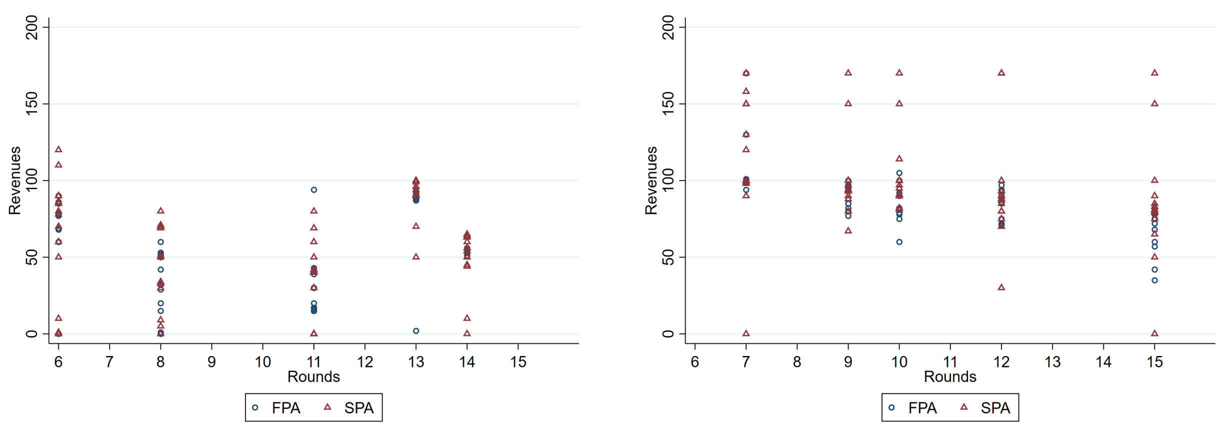
Table 7 provides ordinary least squares (OLS) regression results where the dependant variable is revenue. The explanatory variable "reference v" represents the valuation that appears in the expression of revenue at efficient equilibria. As can be seen in Table 3, in our experiments, the revenue at efficient equilibria is given by $\tilde{v}_2$ in the case of positive externalities

Table 7: Estimation Results for Revenue

Variables	Revenue					
	Positive Externalities			Negative Externalities		
	All	FPA	SPA	All	FPA	SPA
	(1)	(2)	(3)	(4)	(5)	(6)
SPA	2.341			11.02**		
	(3.715)			(3.928)		
reference v	1.000***	1.075***	0.934***	0.517***	0.621***	0.426*
	(0.161)	(0.170)	(0.264)	(0.126)	(0.101)	(0.219)
e	-0.614***	-0.644***	-0.588***	-0.144	-0.149	-0.139
	(0.0896)	(0.116)	(0.136)	(0.134)	(0.109)	(0.232)
Observations	150	70	80	150	70	80
R-squared	0.422	0.541	0.341	0.177	0.461	0.066

Note: Robust Standard errors are in parentheses. The notation *** indicates significance at 1% level, ** at 5% level, * at 10% level.

Figure 2: Revenues



(a) Positive Externalities

(b) Negative Externalities

and v_1 in the case of negative externalities. Thus, we set the reference v as the Blue-High bidder's effective valuation in the case of positive externalities and the Red bidder's valuation in the case of negative externalities.

In column (1) of Table 7, we find that the FPA and the SPA yield similar revenues under positive externalities, but column (4) reveals that the SPA generates higher revenues than the FPA under negative externalities. Hence, only the case of positive externalities is consistent with the prediction about the revenue in Prediction 1. In Figure 2, we observe revenues equal to the maximum 170 coins or an amount close to it in several groups in the SPA under negative externalities, while we observe more concentrated revenues in the other cases. Overbidding in the SPA relative to the FPA occurs more frequently and severely under positive externalities, as shown in Tables 4 and 5, but its effect on the revenue is stronger under negative externalities.

From Table 7, we can also see that a larger size of externalities reduces revenues under positive externalities in columns (1)–(3), but increases revenues under negative externalities in columns (4)–(6) to a lesser degree and insignificantly. Prediction 2 predicts that the revenue decreases in the size of externalities in the case of positive externalities where the Red bidder wins at efficient equilibria while the revenue is independent of the size of externalities in the case of negative externalities. Thus, our results can be considered as consistent with the prediction about the revenue in Prediction 2.

Our results on the revenue can be summarized as follows.

Result 3. *The two auction formats have no difference in revenues under positive externalities, but the SPA yields higher revenues than the FPA under negative externalities. An increase in the level of externalities reduces the revenue under positive externalities and has no effect on it under negative externalities.*

5.1.3 Efficiency

In order to study the effect of the existence of inefficient equilibria on efficiency, we compare the cases where there exist only efficient equilibria with those where there exist inefficient

Table 8: Estimation Results for Efficiency

Variables	Incidence of Efficient Allocations					
	Positive Externalities			Negative Externalities		
	All (1)	FPA (2)	SPA (3)	All (4)	FPA (5)	SPA (6)
SPA	0.041			-0.535**		
	(0.235)			(0.214)		
ineff eqm	-0.322	-0.186	-0.439	-0.721	-0.746	-0.694
	(0.242)	(0.356)	(0.333)	(0.443)	(0.639)	(0.616)
e	0.008	0.013	0.004	-0.015	-0.012	-0.018
	(0.008)	(0.011)	(0.010)	(0.014)	(0.020)	(0.019)
Observations	120	56	64	150	70	80
Log-pseudo likl.	-78.7	-36.3	-41.9	-93.9	-47.3	-46.4

Note: Robust standard errors are in parentheses. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

equilibria as well in a similar environment. For this reason, we drop data from Round 13 in the following analysis of efficiency.

Table 8 shows the Probit regression results in which the dependant variable is the indicator variable for efficient allocations. The explanatory variable "ineff eqm" is the variable indicating the rounds having inefficient equilibria along with efficient equilibria (i.e., Rounds 6 and 8 in the case of positive externalities and Rounds 7, 10, and 15 in the case of negative externalities). Column (1) shows that the two auction formats yield similar proportions of efficient allocations under positive externalities, while column (4) reveals that the SPA treatment reduces efficiency under negative externalities. Hence, our results are consistent with the prediction about the allocation in Prediction 1 only in the case of positive externalities. Our results suggest that overbidding by Blue-Low bidders under negative externalities leads to their winning more frequently in the SPA than in the FPA. This is consistent with our previous observation that overbidding has a stronger effect on revenue in the SPA than in the FPA under negative externalities. We also find no statistical evidence that the existence of inefficient equilibria and the level of externalities affect efficiency, contrary to Prediction

3.

Our results on efficiency can be summarized as follows.

Result 4. *The two auction formats have no difference in achieving efficient allocations under positive externalities, but the FPA achieves efficient allocations more often than the SPA under negative externalities. The existence of inefficient equilibria and the level of externalities have no effect on efficiency.*

5.2 Incomplete Information

In this subsection, we present our results on bidding behavior, revenue, and efficiency in the incomplete information rounds.

In order to test whether bids and allocations converge to those at efficient equilibrium over time, we check whether the incidence of overbids is diminished and that of efficient allocations increases as the rounds progress. Table 9 shows that there are no economically and statistically significant learning effects in these dimensions.

In Table 10, we compare the two auction formats in terms of the incidence of overbids, the revenue, and the incidence of efficient allocations under positive and negative externalities. Columns (1) and (2) show that bidders overbid more frequently in the SPA than in the FPA. Columns (3) and (4) reveal that the revenue is higher in the SPA than in the FPA. Columns (5) and (6) show that efficient allocations arise more often in the FPA than in the SPA, especially under positive externalities. Although these results are qualitatively similar to those on the CI rounds, there are a few noticeable differences. First, revenue is higher in the SPA not only under negative externalities but also under positive externalities, and the magnitudes of the coefficients are quite similar between the two cases. Second, efficiency losses occur in the SPA not under negative externalities as in the CI rounds (although the direction of the effect is still the same as in the CI rounds), but under positive externalities. Although we cannot generalize these differences due to the limited sample size in the II rounds,[12] it seems that private information strengthens overbidding tendencies in the SPA especially

[12]There was only a single set of parameters for each sign of externalities in the II rounds, whereas several parameter sets were used for each sign in the CI rounds.

Table 9: Estimation Results for Learning Effects in the II Rounds

Outcome Var.	Explanatory Var.	Positive Externalities FPA (1)	Positive Externalities SPA (2)	Negative Externalities FPA (3)	Negative Externalities SPA (4)
Overbids	Rounds	0.002	0.015	0.033	0.018
		(0.042)	(0.030)	(0.029)	(0.035)
	Observations	210	270	210	210
	Log-pseudo likl.	-133.0	-184.6	-97.3	-143.8
Efficiency	Rounds	0.068	-0.079	-0.037	-0.008
		(0.054)	(0.052)	(0.056)	(0.054)
	Observations	70	90	70	70
	Log-pseudo likl.	-47.2	-52.8	-44.8	-42.8

Note: Standard errors are clustered at the subject level in Overbids. Robust standard errors are in parentheses in Efficiency. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

Table 10: Estimation Results for Overbids, Revenue, and Efficiency in the II Rounds

Variables	Overbids Positive (1)	Overbids Negative (2)	Revenue Positive (3)	Revenue Negative (4)	Efficiency Positive (5)	Efficiency Negative (6)
SPA	0.612***	1.086***	13.31***	14.87***	-0.700***	-0.120
	(0.218)	(0.289)	(2.336)	(3.091)	(0.206)	(0.221)
Observations	480	420	160	140	160	140
R-squared	-	-	0.151	0.144	-	-
Log-pseudo likl.	-317.7	-241.7	-	-	-102.2	-87.8

Note: Standard errors are clustered at the subject level in Overbids. Robust standard errors are in parentheses in Revenue and Efficiency. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

under positive externalities, thereby leading to higher revenues and efficiency losses in the SPA under positive externalities.

Lastly, we summarize our results on the II rounds as follows.

Result 5. *With incomplete information, there are no learning effects, and there are more overbidding, higher revenues, and less efficient allocations in the SPA than in the FPA.*

6 Concluding Remarks

In this paper, we studied a simple auction setting where there are three bidders and one of the bidders creates positive or negative externalities on the other two bidders. We theoretically and experimentally compared the two well-known auction formats, the FPA and the SPA, in our setting. Using a refinement of undominated Nash equilibria, we derived equilibrium bids and outcomes in the two auction formats under various conditions on the parameters. Based on our theoretical results, we made three predictions for our experiments.

In our experiments, we implemented the two auction formats both with complete information and with incomplete information. In the complete information rounds, we found that participants tend to overbid, especially in the SPA. Although overbidding in the SPA relative to that in the FPA occurred more frequently and severely in rounds with positive externalities, it had stronger effects on the outcomes in rounds with negative externalities. As a result, we observed higher revenues and less efficient allocations in the SPA than in the FPA under negative externalities, while we found no significant differences between the two auction formats in terms of revenues and efficiency under positive externalities. That is, our experimental data were consistent with Prediction 1 only under positive externalities. These findings suggest that standard models are capable of organizing actual bidding behavior and outcomes when externalities are positive, whereas negative externalities seem to require additional elements in the model to enhance its predictive power. Introducing behavioral motives of bidders could be useful in this regard. For instance, negative externalities may affect a participant's emotion more significantly than positive externalities as experimental studies have found that people react more strongly to losses than gains (that is, loss aversion introduced by Kahneman and Tversky, 1979).

Our experimental results were consistent with Prediction 2, which is concerned with the effects of the level of externalities on bids and revenues, especially in the FPA. Lastly, in contrast to Prediction 3, we found no evidence that efficient allocations occur more frequently in rounds where there are only efficient equilibria than in rounds where there are inefficient equilibria as well. In the incomplete information rounds, we also found more overbidding, higher revenues, and lower efficiency in the SPA than in the FPA, and these tendencies persisted even after participants gained experiences.

Although participants do overbid in the FPA, their tendency to overbid is much stronger in the SPA, where we often observe very aggressive bidding behavior such as bidding the maximum amount. Hence, if our goal is to maximize the revenue, the SPA would be a better choice than the FPA. On the other hand, if our goal is to achieve efficient allocations and limit overbidding, the FPA would serve better. In our study, we chose to compare between the FPA and the SPA because they have simple rules that participants can easily understand and are widely used in the real world. In these auction formats, bidders simply choose *one-dimensional* bids. However, in the presence of externalities, a bidder may have different effective valuations against other bidders, and one-dimensional bids are not enough for bidders to convey relevant information about their preferences. Hence, more complicated auction formats that allow *multidimensional* bids (such as the ones proposed by Jehiel et al., 1999 and Jeong, 2019) may perform better, and it would be interesting to theoretically and experimentally compare one-dimensional auction mechanisms with multi-dimensional ones. We leave this topic for future research.

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A Proofs of Propositions

Proof of Proposition 1. (1) Consider the FPA. We first show that bidder $j \neq 1$ cannot obtain the object at any Nash equilibrium. Suppose to the contrary that bidder $j \neq 1$ obtains the object at a Nash equilibrium (b_1, b_2, b_3). Then $b_j \leq v_j < v_1$, and bidder 1 can gain by deviating to $b_1 \in (b_j, v_1)$. Hence, bidder 1 obtains the object at any Nash equilibrium, and a bid profile (b_1, b_2, b_3) is a Nash equilibrium if and only if $b_1 \in [\tilde{v}_2, v_1]$, $b_1 \geq b_j$ for all $j \neq 1$, and $b_1 = b_j$ for some $j \neq 1$, assuming that ties are broken in favor of bidder 1. Then bidders 2 and 3 compete with bidder 1 at any Nash equilibrium, and thus (b_1, b_2, b_3) is an effectively undominated Nash equilibrium if and only if $b_1 = b_2 = \tilde{v}_2$ and $b_3 \leq \tilde{v}_3$.

(2) Consider the SPA. Bidder 1 has a weakly dominant strategy $b_1 = v_1$. For bidder $j \neq 1$, $b_j < \tilde{v}_j$ and $b_j > v_j$ are weakly dominated by $b_j = \tilde{v}_j$ and $b_j = v_j$, respectively. Hence, if a bid profile (b_1, b_2, b_3) is an undominated Nash equilibrium, then $b_1 = v_1$ and $b_j \in [\tilde{v}_j, v_j]$ for all $j \neq 1$. Since bidder 1 is the highest bidder at any undominated Nash equilibrium, $(b_1, b_2, b_3) = (v_1, \tilde{v}_2, \tilde{v}_3)$ is the unique effectively undominated Nash equilibrium.

Proof of Proposition 2. (1) Consider the FPA. Let us assume that ties are broken in favor of bidder 1. Suppose that there is a Nash equilibrium (b_1, b_2, b_3) where bidder 1 obtains the object. In order to prevent deviations by bidders 1 and 2, we need to have $b_1 \leq v_1$ and $e \geq v_2 - b_1$, respectively, which implies $v_1 \geq \tilde{v}_2$. Suppose that $v_1 \geq \tilde{v}_2$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 1 obtains the object if and only if $b_1 = b_2 = \tilde{v}_2$ and $b_3 \leq \tilde{v}_3$.

Let us assume that ties are broken in favor of bidder 2. Suppose that there is an effectively undominated Nash equilibrium (b_1, b_2, b_3) where bidder 2 obtains the object. Then $b_2 = b_j$ for some $j \neq 2$. Suppose that $b_2 = b_1$. Then we need to have $b_2 = b_1 = v_1 \leq \tilde{v}_2$ and $v_1 > v_3 \geq b_3$. Suppose that $b_2 = b_3$. Then we need to have $b_2 = b_3 = v_3 < v_2$ and $v_3 > v_1 \geq b_1$. Thus, we obtain $\tilde{v}_2 \geq v_1 > v_3$ or $v_3 > v_1$. Suppose that $\tilde{v}_2 \geq v_1 > v_3$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $b_2 = b_1 = v_1$ and $b_3 \leq v_3$. Note that bidder 2 has no incentive to deviate because $v_2 - b_2 \geq e$. Suppose that $v_3 > v_1$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $b_2 = b_3 = v_3$ and $b_1 \leq v_1$.

Suppose that bidder 3 obtains the object at a Nash equilibrium (b_1, b_2, b_3). Then $b_3 \leq v_3$. Since $v_2 > v_3$, bidder 2 can gain by deviating to $b_2 \in (b_3, v_2)$. Hence, there is no Nash equilibrium where bidder 3 obtains the object.

(2) Consider the SPA. Let us assume that ties are broken in favor of bidder 1. At any undominated Nash equilibrium, bidder 1 chooses $b_1 = v_1$. At any effectively undominated Nash equilibrium where bidder 1 obtains the object, bidders 2 and 3 compete with bidder 1 and thus choose $b_j = \tilde{v}_j$ for all $j \neq 1$. Hence, there is an effectively undominated Nash equilibrium where bidder 1 obtains the object if and only if $v_1 \geq \tilde{v}_2$. If such an equilibrium exists, it is given by $(b_1, b_2, b_3) = (v_1, \tilde{v}_2, \tilde{v}_3)$, and bidder 1 pays the second highest bid $\tilde{v}_2$ at the equilibrium.

Let us assume that ties are broken in favor of bidder 2. At any effectively undominated Nash equilibrium where bidder 2 obtains the object and competes with bidder 1, bidder 2 chooses $b_2 = \tilde{v}_2$ and bidder 3 chooses $b_3 = v_3$. Hence, there is an effectively undominated Nash equilibrium where bidder 2 obtains the object and competes with bidder 1 if and only if $\tilde{v}_2 \geq v_1 > v_3$. If such an equilibrium exists, it is given by $(b_1, b_2, b_3) = (v_1, \tilde{v}_2, v_3)$, and bidder 2 pays the second highest bid v_1 at the equilibrium. At any effectively undominated Nash equilibrium where bidder 2 obtains the object and does not compete with bidder 1, bidders 2 and 3 choose $b_j = v_j$ for all $j \neq 1$. Hence, there is an effectively undominated Nash equilibrium where bidder 2 obtains the object and does not compete with bidder 1 if and only if $v_3 > v_1$. If such an equilibrium exists, it is given by $(b_1, b_2, b_3) = (v_1, v_2, v_3)$, and bidder 2 pays the second highest bid v_3 at the equilibrium.

Suppose that bidder 3 obtains the object at an effectively undominated Nash equilibrium (b_1, b_2, b_3). Then b_3 is either $\tilde{v}_3$ or v_3 depending on whether bidder 3 competes with bidder

1 or not. Since $v_2 > v_3 > \tilde{v}_3$, bidder 2 can gain by deviating to $b_2 > b_3$. Hence, there is no effectively undominated Nash equilibrium where bidder 3 obtains the object.

Proof of Proposition 3. (1) Consider the FPA. Suppose that there is a Nash equilibrium (b_1, b_2, b_3) where bidder 1 obtains the object. In order to prevent deviations by bidders 1 and 2, we need to have $b_1 \leq v_1$ and $e \geq v_2 - b_1$, respectively. Since $v_2 > v_1$ and $e < 0$, the two inequalities cannot be satisfied simultaneously, which is a contradiction. Hence, there is no Nash equilibrium where bidder 1 obtains the object.

Suppose that there is an effectively undominated Nash equilibrium (b_1, b_2, b_3) where bidder 3 obtains the object. Suppose that bidder 3 does not compete with bidder 1 at (b_1, b_2, b_3). Then $b_3 \leq v_3$. Since $v_2 > v_3$, bidder 2 can gain by deviating to $b_2 \in (b_3, v_2)$. Suppose that bidder 3 competes with bidder 1 at (b_1, b_2, b_3). Then $b_3 = b_1 \leq v_1$. Since $v_2 > v_1$, bidder 2 can gain by deviating to $b_2 \in (b_3, v_2)$. In either case, we obtain a contradiction. Hence, there is no effectively undominated Nash equilibrium where bidder 3 obtains the object.

Let us assume that ties are broken in favor of bidder 2, and we look for effectively undominated Nash equilibria where bidder 2 obtains the object. Suppose that $v_1 > v_3$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $b_2 = b_1 = v_1$ and $b_3 \leq v_3$. Suppose that $v_3 > v_1$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $b_2 = b_3 = v_3$ and $b_1 \leq v_1$.

(2) Consider the SPA. Bidder 1 has a weakly dominant strategy $b_1 = v_1$. For bidder $j \neq 1$, $b_j < v_j$ and $b_j > \tilde{v}_j$ are weakly dominated by $b_j = v_j$ and $b_j = \tilde{v}_j$, respectively. Hence, if a bid profile (b_1, b_2, b_3) is an undominated Nash equilibrium, then $b_1 = v_1$ and $b_j \in [v_j, \tilde{v}_j]$ for all $j \neq 1$. We have $b_2 > b_1$ for any $b_2 \in [v_2, \tilde{v}_2]$, and thus bidder 1 cannot obtain the object at any undominated Nash equilibrium. Consider any effectively undominated Nash equilibrium (b_1, b_2, b_3) where bidders 2 and 3 compete with each other. Then we have $b_2 = v_2$ and $b_3 = v_3$. In order to have $b_1 = v_1$ as the lowest bid, we should have $v_3 > v_1$. Consider any effectively undominated Nash equilibrium (b_1, b_2, b_3) where bidder $j \neq 1$ competes with bidder 1. Then we have $b_j = \tilde{v}_j$ and $b_k = v_k$ for $k \neq 1, j$. In order to have $b_1 = v_1$ as the second highest

bid, we should have $v_1 > v_3$ and $(j,k) = (2,3)$. If $v_1 > v_3$, then $(b_1, b_2, b_3) = (v_1, \tilde{v}_2, v_3)$ is the unique effectively undominated Nash equilibrium. If $v_3 > v_1$, then $(b_1, b_2, b_3) = (v_1, v_2, v_3)$ is the unique effectively undominated Nash equilibrium.

Proof of Proposition 4. Part (i) can be proven as in the proof of Proposition 2, and we prove parts (ii) and (iii) in the following.

(1) Consider the FPA. Let us assume that ties are broken in favor of bidder 2. Suppose that there is a Nash equilibrium (b_1, b_2, b_3) where bidder 2 obtains the object. Then $b_2 = b_j$ for some $j \neq 2$. Suppose that $b_2 = b_3$. In order to prevent a deviation by bidder 2, we need to have $b_2 \leq v_2$. Since $v_1 > v_2$, bidder 1 can gain by deviating to $b_1 \in (b_2, v_1)$, a contradiction. Hence, it must be that $b_2 = b_1$. In order to prevent deviations by bidders 1 and 2, we need to have $b_2 \geq v_1$ and $v_2 - b_2 \geq e$, respectively. Combining these two inequalities, we obtain $\tilde{v}_2 \geq v_1$. Now suppose that $\tilde{v}_2 \geq v_1$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $b_2 = b_1 = v_1$ and $b_3 \leq v_3$.

Let us assume that ties are broken in favor of bidder 3. Suppose that there is a Nash equilibrium (b_1, b_2, b_3) where bidder 3 obtains the object. Then $b_3 = b_j$ for some $j \neq 3$. Suppose that $b_3 = b_2$. In order to prevent a deviation by bidder 3, we need to have $b_3 \leq v_3$. Since $v_1 > v_3$, bidder 1 can gain by deviating to $b_1 \in (b_3, v_1)$, a contradiction. Hence, it must be that $b_3 = b_1$. In order to prevent deviations by bidders 1 and 3, we need to have $b_3 \geq v_1$ and $v_3 - b_3 \geq e$, respectively. Combining these two inequalities, we obtain $\tilde{v}_3 \geq v_1$. Now suppose that $\tilde{v}_3 \geq v_1$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 3 obtains the object if and only if $b_3 = b_1 = v_1$ and $b_2 \leq v_2$. Since $b_3 = v_1 > v_2$, bidder 2 has no incentive to deviate.

(2) Consider the SPA. Let us assume that ties are broken in favor of bidder 2. At any effectively undominated Nash equilibrium where bidder 2 obtains the object and competes with bidder 1, bidder 2 chooses $b_2 = \tilde{v}_2$ and bidder 3 chooses $b_3 = v_3$. Since $v_1 > v_3$, there is an effectively undominated Nash equilibrium where bidder 2 obtains the object and competes with bidder 1 if and only if $\tilde{v}_2 \geq v_1$. If such an equilibrium exists, it is given by $(b_1, b_2, b_3) = (v_1, \tilde{v}_2, v_3)$, and bidder 2 pays the second highest bid v_1 at the equilibrium. At any effectively

undominated Nash equilibrium where bidder 2 obtains the object and does not compete with bidder 1, bidders 2 and 3 choose $b_j = v_j$ for all $j \neq 1$. Since $v_1 > v_2 > v_3$, there is no such equilibrium.

Since $v_1 > v_2$, part (iii) for the SPA can be proven as in the proof of part (ii) above.

B Experimental Instructions

Thank you for participating in the experiment. Please read the following instructions carefully.

Your decisions will be anonymously collected and used only for research. No one will know what your decisions are in the experiment.

You will obtain a gift certificate worth KRW 5,000 as a show-up fee. In addition to this show-up fee, you can earn an additional gift certificate whose value depends on your decisions as well as your group members' in the experiment.

You will participate in an auction for an item. Here is the rule:

- You are randomly grouped with others in this room to form a group of three. (Members do not know each other's identity.) The three members in a group participate in the auction. In each group, one member is called Red and the other two are called Blue.
- 170 coins are in your virtual account. You choose how many coins to bid out of 170 coins you have. The member with the highest bid wins. If there is more than one bidder who submits the highest bid, the winner is randomly chosen with equal chances.
- **[For the FPA treatment]** If you win, V coins are added to your account. (V can be different across members.) You pay your bid. In your account: 170 + V - [your bid].
- **[For the SPA treatment]** If you win, V coins are added to your account. (V can be different across members.) You pay the second highest bid. If there is more than one bidder who submits the highest bid, the second highest bid is equal to the highest bid. In your account: 170 + V - [the second highest bid].

- If you lose:
  - If you are Blue and the winner is Red, E coins are added to your account. (E is the same for both Blue members.) If $E < 0$, this means that coins are subtracted from your account. In your account: $170 + E$.
  - If you are Blue and the winner is another Blue, no change is made to your account. In your account: 170.
  - If you are Red, no change is made to your account. In your account: 170.

You will play this auction for 25 rounds.

- The first 5 rounds are for practice. In each round, you are randomly re-grouped with others. These 5 practice rounds are not considered for payments. You have two minutes to make your decision in each round. (If you do not make your decision within two minutes, 0 will be entered as your decision.)
- The next 20 rounds are considered for payments. You have one minute to make your decision in each round.
  - For the first 10 rounds, you are randomly re-grouped with others in each round. Everyone knows the value of E and all the members' values of V.
  - For the second 10 rounds, you are randomly grouped with others in the first round and then the group stays the same for the 10 rounds. Everyone knows the value of E and his/her own value of V but not the others' values of V. That is, you do not know how many coins your group member obtains when he/she wins the auction. The value of V, which is fixed throughout the 10 rounds, is an integer between 30 and 100, and it can be different across your group members.

After the 25 rounds of auctions, the experiment ends. From the 20 rounds considered for payments, one round will be chosen randomly, and the total amount of your coins in that round will be converted to KRW 95 per coin and given to you as a gift certificate (in addition to your show-up fee).

Please do not talk with others nor use your phones. Please take your time when making your decisions in the experiment; you do not have to hurry.

If you have any question, please raise your hand. Please wait for further instructions.