The Heterogeneous Welfare Effects of Business Cycles*

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October 2022

Abstract

This study investigates the welfare effects of business cycle fluctuations from a distributional perspective. To this end, we develop a quantitative heterogeneous-agent model which incorporates market incompleteness and non-convexity into the mapping from the time devoted to work to labor services. In this setup, households can insure against aggregate uncertainty using labor and savings and have substantially different labor supply elasticities. We find that the welfare effects are heterogeneous across households, with wealth-rich households benefiting most from business cycles. Wealth-rich households enjoy business cycles more than wealth-poor households, because they experience less volatile consumption and can enjoy higher average income through reallocating savings intertemporally.

Key Words: Business Cycle, Welfare, Inequality, Volatility Effect, Level Effect, Non-convexity

JEL Classification: E32, D61, D31

*We would like to thank Byoungchan Lee, Joonseok Oh, and Choongryul Yang for their valuable comments and suggestions. All errors are ours.

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1 Introduction

How business cycles affect consumers' welfare in the economy has long been an important topic in the business cycle literature. The seminal contribution by Lucas (1987) suggests that eliminating business cycle uncertainty has a negligible effect on economic welfare, as the welfare cost of business cycle fluctuations is less than 0.01 percent of average consumption. Since then, there have been numerous studies on this issue to understand whether other realistic features of the economy change Lucas's estimate. However, most of these studies have mainly focused on the average welfare effect, using quantitative models with a homogeneous representative household.[1]

In recent years, increasing inequality has been a primary concern of economists and policymakers, spurring extensive research on disaggregate effects of business cycles.[2] While the majority of this strand of research focuses on the positive effects, much less attention has been paid to normative questions. The main objective of this study is to fill the gap in the literature by assessing the welfare effects of business cycle fluctuations from a disaggregate perspective.

We consider a heterogeneous-agent real business cycle (RBC) model in which individual households are subject to idiosyncratic shocks to labor productivity (as in Aiyagari, 1994) and time discount factors (as in Krusell and Smith, 1998). Aggregate productivity shocks are introduced as a driving force of the business cycle. Moreover, we introduce realistic assumptions into both the labor and asset markets. Following Rogerson and Wallenius (2009) and Chang et al. (2019), we consider the operative intensive and extensive margins of labor supply by introducing non-convexity into the mapping from hours worked to labor services. The non-convexity mapping generates substantial heterogeneity in labor supply elasticity across households, capturing countercyclical income inequality in the data.[3] Regarding the asset market, we assume incomplete asset markets in which households cannot fully insure against idiosyncratic shocks and face borrowing constraints. These features together allow our calibrated model to successfully reproduce cross-sectional distributions

[1] For example, see Tallarini (2000), Otrok (2001), Alvarez and Jermann (2004), Barlevy (2004), Lester, Pries and Sims (2014) and Cho, Cooley and Kim (2015).

[2] Specifically, the heterogeneous effects of monetary and fiscal policy have been at the center of recent academic research. Important contributions include Gornemann, Kuester and Nakajima (2021), Kaplan, Moll and Violante (2018), and Auclert (2019) for monetary policy and Kaplan and Violante (2014) and Ma (2019) for fiscal policy.

[3] Castaneda, Díaz-Giménez and Ríos-Rull (1998) and Kwark and Ma (2021) show that the extensive margin is an important driver of income inequality over the business cycle.

of income, earnings, and wealth and business cycle statistics in the U.S. data. In this environment, business cycle fluctuations affect households' welfare differently depending on i) their sources of income and ii) how well they use labor and savings in the face of aggregate risks. Thus, our model highlights an interesting interaction between the market incompleteness and endogenous factor supply when accounting for the welfare effects of business cycles at the micro level.

The main result of the paper is as follows. The welfare effects are heterogeneous across households, with wealth-rich households enjoying business cycles more than wealth-poor households. To understand the reason for this result, we decompose the total welfare effect into two components: volatility and level effects. The volatility effect captures the cost of consumption volatility for risk-averse households (Lucas, 1987) and the benefit of households making use of the uncertainty in their favor by adjusting factor inputs intertemporally. Intertemporal adjustment of factor inputs such as labor and savings can be beneficial because households can work and save more (less) in periods of high (low) aggregate productivity and thus increase the mean income. The level effect is related to the fact that business cycles can increase welfare by raising the long-run average factor inputs and thus the mean output through households' precautionary motives (Cho, Cooley and Kim, 2015). Interestingly, we find that volatility and level effects are inversely related in the dimension of wealth: the former increases with wealth, whereas the latter decreases with wealth. This result implies that the increasing pattern of the total welfare effects by wealth mainly arises from the volatility effects. That is, wealth-rich households enjoy business cycles more than wealth-poor households, because they experience less volatile consumption and can enjoy higher average income by reallocating savings intertemporally.

Related Literature

Significant research has attempted to understand whether various features of the economy result in a higher cost of business cycle fluctuation than what Lucas (1987) has estimated. Using fluctuations in asset prices, Alvarez and Jermann (2004) infer the costs of consumption fluctuations and obtain much larger estimates for the welfare cost. Similarly, Tallarini (2000) considers recursive utility functions and finds that the welfare costs of business cycles can be high if preference parameters are set to target asset prices. Otrok (2001) finds that time-non-separabilities in preferences that match the key business cycle statistics lead to a small welfare cost of business cycles. Barlevy (2004) argues that the welfare cost is likely to be substantial when fluctuations affect the growth rate of consumption. All of these papers only consider the costs of consumption fluctuations for risk-averse households, ignoring the level effects that we consider, and so find that business cycles are always welfare-reducing.

One strand of literature argues that business cycle fluctuations are not always detrimental to households' welfare using representative-agent models. Lester, Pries and Sims (2014) and Cho, Cooley and Kim (2015) show that business cycles can increase welfare when factor supply is sufficiently elastic, offsetting the costs of consumption fluctuations emphasized by Lucas (1987) and others mentioned above. Specifically, Cho, Cooley and Kim (2015) decompose the total welfare effect of business cycles into the volatility and level effects and argue that the welfare cost estimates based only on the cost of consumption fluctuations can be seriously biased.[4] Instead, if one considers the level effect, the total welfare effect of business cycle fluctuations can be positive.[5] Lester, Pries and Sims (2014) show that the results of Cho, Cooley and Kim (2015) are robust to the inclusion of variable capital utilization, investment-specific productivity shocks, and preference shocks. The current paper contributes to this literature by considering heterogeneous households with endogenous labor and capital choices in an incomplete markets model. Our model allows us to estimate the distribution of welfare effects of business cycle fluctuations and analyze the relationship between the volatility and level effects at the individual household level.

This paper is also related to the literature on the welfare effect of aggregate uncertainty in heterogeneous-agent models. Schulhofer-Wohl (2008) considers a complete markets environment in which people have different degrees of risk aversion and find that the insurance trade reduces the welfare cost of business cycles for everyone. In contrast, our model assumes incomplete markets. Storesletten, Telmer and Yaron (2001), De Santis (2007), and Krebs (2007) estimate sizable costs of business cycles in an incomplete markets economy with cyclical idiosyncratic risk. In contrast to the present paper, they focus on the volatility effect and do not consider the level effects when computing the welfare effects of business cycles. The work of Krusell et al. (2009) is the closest to ours in that they work on

[4]Even though Cho, Cooley and Kim (2015) use different names for the two effects–fluctuation and mean effects–their decomposition is conceptually similar to ours.

[5]Heiberger and Maussner (2020) show that the level effect computed by Cho, Cooley and Kim (2015) can be systematically overestimated due to a limitation of standard second-order solutions. However, our results are not affected by this argument, since we solve our model using a global method.

the same topic in an incomplete markets production economy with infinitely-lived households. They find welfare losses from business cycles, rather than gains. This is because they do not incorporate endogenous labor choice, which helps to improve welfare by reducing consumption fluctuations. Moreover, they underestimate the level effect, which is an important component that increases the welfare effect in our work.[6]

The rest of the paper is organized as follows. In Section 2, we develop the baseline model and describe the parameter values. Section 3 assesses the extent to which our model reproduces the empirical cross-sectional distributions, the labor supply elasticity, and the business cycle statistics. Section 4 examines the welfare effects of business cycle fluctuations for individual households. In Section 5, we investigate the role of each model ingredient in shaping the welfare effects. Section 6 concludes.

2 Model

We extend the model of Chang et al. (2019), which features intensive and extensive margins of labor supply, by incorporating heterogeneity in the discount factor as in Krusell and Smith (1998). The economy is populated by households that face two types of uninsurable idiosyncratic shocks: the idiosyncratic productivity shocks and the discount factor shocks. Asset markets are incomplete, so households must self-insure against idiosyncratic shocks by saving in physical capital and by working. These two types of idiosyncratic shocks are essential for the model to generate the realistic distribution of wealth and earnings as well as that of hours worked.

2.1 Setup

There is a unit measure of ex-ante identical infinitely lived individuals, indexed by i. Each individual chooses streams of consumption, $c_{i,t}$, and the time devoted to work, $h_{i,t} \in [0, 1]$, that maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} B_t \left[\ln c_{i,t} - \chi \frac{h_{i,t}^{1+1/\nu}}{1+1/\nu} \right],$$

[6]Using a model with endogenous labor supply and idiosyncratic wage risk, the seminal work by Heathcote, Storesletten and Violante (2008) explores the welfare effect of a rise in cross-sectional wage dispersion in an economy without aggregate shocks and capital.

subject to

$$c_{i,t} + a_{i,t+1} = w_t z_{i,t} g(h_{i,t}) + (1 + r_t) a_{i,t} \tag{1}$$

and $a_{i,t+1} \geq \underline{a}$, where χ is the parameter that governs the disutility from working, and ν is the curvature parameter for hours worked. As in Kaplan, Moll and Violante (2018), we assume that the borrowing limit, $\underline{a} < 0$, is equal to the model-implied average labor income. $a_{i,t+1}$ denotes the claims to physical capital that individuals trade, r_t is the rate of return to capital, and w_t is the real wage rate per effective unit of labor. $B_t = \Pi_{s=0}^{t} \beta_s$ denotes the cumulative idiosyncratic discount factor between period 0 and t. The time discount factor, β, can take on two values, i.e., $\beta \in \{\beta_L, \beta_H\}$, where $0 < \beta_L < \beta_H < 1$. The time discount factor evolves according to the Markov process with transition probability $\pi_\beta(\beta'|\beta)$, which will be discussed later.

Idiosyncratic productivity, denoted by z_{it}, evolves according to:

$$\ln z_{i,t} = \rho_z \ln z_{i,t-1} + \varepsilon_{z,t} \qquad \text{with} \qquad \varepsilon_{z,t} \overset{iid}{\sim} \mathcal{N}(0, {\sigma_z}^2),$$

where ρ_z is the persistence of idiosyncratic productivity. $g(h_{i,t})$ is the units of labor services that are used for production. Our model distinguishes between the extensive and intensive margins of labor supply, as the employment decision is known to be a crucial driver of inequality dynamics over business cycles (Castaneda, Díaz-Giménez and Ríos-Rull, 1998; Kwark and Ma, 2021). To generate both margins of labor supply endogenously, we embed a non-convexity into the mapping from the time devoted to work into units of labor services, adopted in Rogerson and Wallenius (2009) and Chang et al. (2019):

$$g(h_{i,t}) = \max\{h_{i,t} - \Delta, 0\},$$

where $0 < \Delta < 1$ is a fixed cost of devoting time to work. Interpretations of this fixed cost would include the time spent for supervising or coordinating with other workers. If an individual with idiosyncratic productivity $z_{i,t}$ devotes $h_{i,t}$ units of time to work, the individual will earn $w_t z_{i,t} g(h_{i,t})$. Rich *ex-post* heterogeneity across households and the non-convexity mapping will cause individual households to behave heterogeneously over the business cycle, which will affect households' welfare differently.

There is a representative firm that produces aggregate output using aggregate labor services, N_t, and capital, K_t, according to a Cobb–Douglas production function:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}, \tag{2}$$

where α denotes the capital share. Output is used for either consumption or investment, and capital depreciates at rate δ in each period. A_t is the aggregate productivity, which evolves according to

$$A_t = (1-\rho_A) + \rho_A A_{t-1} + \varepsilon_{A,t} \qquad \text{with} \qquad \varepsilon_{A,t} \overset{iid}{\sim} \mathcal{N}(0, {\sigma_A}^2),$$

where ρ_A is the persistence of aggregate productivity. Note that we follow Lester, Pries and Sims (2014) and specify the exogenous process for A_t in levels instead of log-levels to eliminate the effect of Jensen's inequality.[7]

2.2 Equilibrium

To define the equilibrium recursively, we need to construct the value function. The individual state variables are capital, a, the discount factor, β, and labor productivity, z. The aggregate state variables are aggregate productivity, A, and a measure, μ, over the individual state variables. The value function for an individual household is:

$$V(a,\beta,z;A,\mu) = \max_{c,a',h} \left\{ \ln c - \chi \frac{h^{1+1/\nu}}{1+1/\nu} + \mathbb{E}[\beta V(a',\beta',z';A',\mu')|z,\beta,A] \right\} \tag{3}$$

subject to

$$\begin{aligned} &c = \max\{0, h-\Delta\} w(A,\mu) z + (1+r(A,\mu))a - a' \\ &c \geq 0, \quad a' \geq \underline{a}, \quad 0 \leq h \leq 1 \\ &\mu' = \Gamma(A,\mu), \end{aligned}$$

where $\Gamma(.)$ is the equilibrium law of motion for μ. The recursive competitive equilibrium consists of individual decision rules $\{c(a,\beta,z;A,\mu), a'(a,\beta,z;A,\mu), h(a,\beta,z;A,\mu)\}$, a value function $V(a,\beta,z;A,\mu)$, factors of production $\{K(A,\mu), N(A,\mu)\}$, factor prices $\{r(A,\mu), w(A,\mu)\}$, and a law of motion $\Gamma(A,\mu)$, such that

[7] If we wrote the process for A_t as an AR(1) in log-levels, the mean A_t would mechanically increase as the innovation variance rises due to Jensen's inequality (Lester, Pries and Sims, 2014).

1. *Given the factor prices, individual decision rules solve value function* (3)*;*

2. *The representative firm maximizes profits:* $r(A,\mu) = F_1(K(A,\mu), N(A,\mu), A) - \delta$ *and* $w(A,\mu) = F_2(K(A,\mu), N(A,\mu), A)$*;*

3. *The goods market clears:* $\int c(a,\beta,z;A,\mu)d\mu + \int a'(a,\beta,z;A,\mu)d\mu = Y + (1-\delta)K$*;*

4. *Factor markets clear:*
   $K'(A,\mu) = \int a'(a,\beta,z;A,\mu)d\mu$ *and* $N(A,\mu) = \int zg(h(a,\beta,z;A,\mu))d\mu$*;*

5. *Individual and aggregate behaviors are consistent: For all* $B_a \subset \mathcal{B}_a$*,* $B_\beta \subset \mathcal{B}_\beta$*, and* $B_z \subset \mathcal{B}_z$*,*

$$\mu'(B_a, B_\beta, B_z) = \int_{B_a,B_\beta,B_z} \left\{ \int_{\mathcal{B}_a,\mathcal{B}_\beta,\mathcal{B}_z} \mathbf{1}\{a' = a'(a,\beta,z;A,\mu)\}\pi_\beta(\beta'|\beta)\pi_z(z'|z)d\mu \right\} da'd\beta'dz',$$

where $\pi_z(z'|z)$ is the transition probability across the idiosyncratic productivity states.

2.3 Calibration

The model frequency is one quarter. There are two sets of parameters: One set jointly targets multiple moments, and the other set targets a single moment one-to-one. Regarding the set of parameters that jointly match multiple moments, we search for 7 parameters that solve 7 restrictions. Specifically, the discount factors, β_H and β_L, the labor disutility parameter, χ, the fixed cost, Δ, the borrowing limit, $\underline{a}$, the transition probability between the highest idiosyncratic productivity levels, $\pi_z(z_{11}|z_{11})$, and the standard deviation of idiosyncratic productivity shock, σ_z, are jointly chosen to match the following targets:[8] the top 5 percent earnings share (SCF 1992), wealth Gini of 0.79 (SCF 1992), earnings Gini of 0.61 (SCF 1992), quarterly average labor income which equals $\underline{a}$, the annual return on capital of 4 percent, the employment rate of 77 percent (SCF 1992), and the average hours worked conditional on working of 0.25 (PSID 1992). These parameters are listed in the *internal calibration* part of Table 1.

[8] We discretize the AR(1) process for idiosyncratic productivity to an eleven-point Markov chain using the Tauchen (1986) method, which gives the transition probability matrix across the idiosyncratic productivity states. We then choose $\pi_z(z_{11}|z_{11})$ and then normalize $\pi_z(z'|z_{11})$ for all $z' \in \{z_1, ..., z_{10}\}$ so that $\sum_{z'} \pi_z(z'|z_{11}) = 1$ holds.

Table 1. Calibration of the Parameters

Symbol	Description	Value	Target (Source)
	Internal calibration		
β_H	High discount factor	0.9833	See text
β_L	Low discount factor	0.9440	See text
χ	Disutility parameter	15.9	See text
Δ	Time fixed cost	0.12	See text
$\underline{a}$	Borrowing limit	-0.4	See text
$\pi_z(z_{11}\|z_{11})$	z_{11} to z_{11} transition probab.	0.94	See text
σ_z	Std. of idiosyncratic prod. shock	0.245	See text
	External calibration		
ν	Curvature parameter	1	Chang et al. (2019)
δ	Capital depreciation rate	0.025	Standard
α	Capital income share	0.33	Standard
ρ_z	AR(1) of idiosyncratic prod. shock	0.929	Chang and Kim (2007)
ρ_A	AR(1) of aggregate prod. shock	0.95	Standard
σ_A	Std. of aggregate prod. shock	0.0103	Output volatility

The following parameters are calibrated externally. As in Chang et al. (2019) and Ma (2022*a*), the curvature parameter for hours worked, ν, is set to 1. We use the AR(1) coefficient of idiosyncratic productivity shock 0.929, which is estimated by Chang and Kim (2007). Regarding the transition matrix of the discount factor, as in Gornemann, Kuester and Nakajima (2021), we assume that it is symmetric, i.e., $\pi_\beta(\beta_L|\beta_L) = \pi_\beta(\beta_H|\beta_H)$. We choose $\pi_\beta(\beta_L|\beta_L)$ to target the average duration of discount factors of 40 years.[9] Given $\pi_\beta(\beta_L|\beta_L)$ and $\pi_\beta(\beta_H|\beta_H)$, $\pi_\beta(\beta_H|\beta_L)$ and $\pi_\beta(\beta_L|\beta_H)$ can be obtained, using the fact $\pi_\beta(\beta_L|\beta_L) + \pi_\beta(\beta_H|\beta_L) = 1$ and $\pi_\beta(\beta_H|\beta_H) + \pi_\beta(\beta_L|\beta_H) = 1$. We assume that the AR(1) coefficient of the aggregate productivity, ρ_A, and its standard deviation, σ_A, are 0.95 and 0.0103, respectively. The standard deviation, σ_A, is chosen to match the empirical standard deviation of HP-filtered output over the period 1967Q1-2020Q4. The capital income share, α, and the capital depreciation rate, δ, are set to 0.33 and 0.025, respectively. These parameters are listed in the *external calibration* part of Table 1.

[9]The idea of introducing a stochastic discount factor is to capture the changes in the saving behavior between generations. The average duration of 40 years for each discount factor is chosen to roughly match the length of a generation.

3 Model Properties

Before we discuss the welfare implication of our model, in this section, we discuss how well the model replicates the cross-sectional distribution, the labor supply elasticity, and the business cycle dynamics in the U.S. data.

3.1 Cross-Sectional Distribution and Labor Supply Elasticity

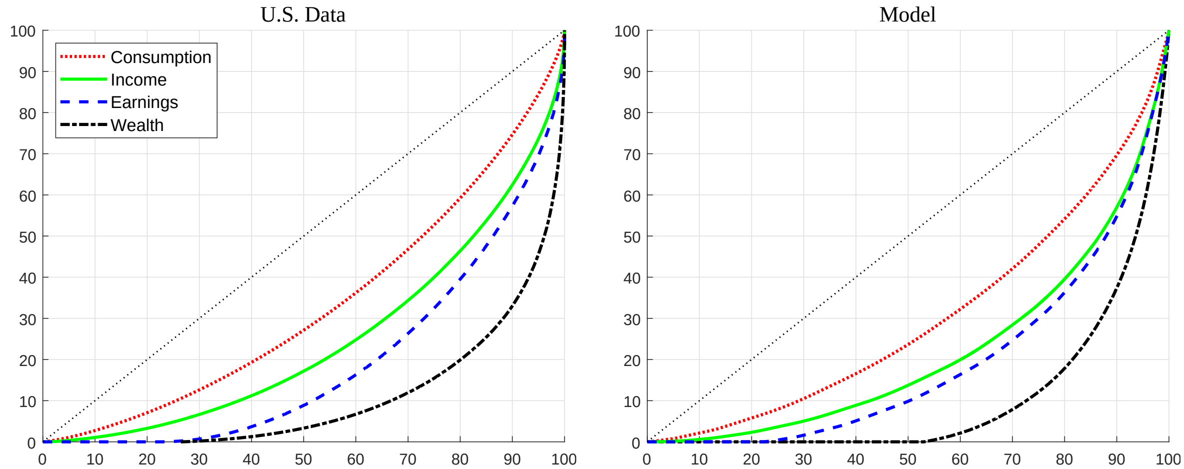
Cross-sectional distribution Given the parameter values that we discussed in the previous section, we compare the steady-state distribution predicted from our model and the distribution in the U.S. data. Figure 1 plots the Lorenz curve in the data (left panel) and the model (right panel) for consumption, income, earnings, and wealth.[10] The wealth distributions in the model and data are similar in that the top 20 percent in wealth hold 80 percent of the total wealth. The top 20 percent hold approximately 60 percent of the total earnings in both the model and data. This is not surprising, considering that the wealth and earnings Gini coefficients are targeted moments. Although the distributions of income and consumption are not targeted, their distributions in the model and data are comparable. The Gini coefficients for income and consumption in the model economy are 0.56 and 0.38, respectively, which are very similar to those in the data (0.51 and 0.33, respectively). Regarding the right tail of the distribution, Table 2 reveals that our model reproduces the top 5 percent income, earnings, and wealth share and the top 1 percent income and earnings share quite well.[11] In sum, we find that our model does a good job of replicating the empirical cross-sectional distributions found in the U.S. data.[12]

[10]The data for income, earnings, and wealth, used for plotting the Lorenz curve, are obtained from the SCF 1992, while the data for consumption are from the Consumer Expenditures Survey (CEX) in 1992. In the data, income is the sum of wages, self-employment and business income, taxable and tax-exempt interest, dividends, realized capital gains, food stamps and other support programs provided by the government, pension income and withdrawals from retirement accounts, social security income, alimony and other support payments, and miscellaneous sources of income. Earnings are wages and salaries. Wealth is the net worth of the households. In the model, income is defined as the sum of labor and capital incomes. Earnings correspond to labor income, while wealth is capital held by households. In both the data and model, consumption is the expenditure on non-durable goods.

[11]Our model underpredicts the top 1 percent of the wealth distribution. In fact, this is a well-known challenge for standard heterogeneous-agent business cycle models unless one adds an additional element. For instance, Benhabib, Bisin and Luo (2019) and Guvenen et al. (2019) show that introducing the rate of return heterogeneity in wealth is a powerful modeling tool that can generate the top 1 percent of the wealth distribution.

[12]In Appendix B.1, we study the role of idiosyncratic productivity shocks and discount factor shocks in shaping the distribution at the steady state.

Figure 1. Cross-Sectional Distribution



Note: Lorenz curve in the U.S. data and model. The data for income, earnings, and wealth, used for plotting the Lorenz curve, are obtained from the SCF 1992, while the data for consumption are from the CEX 1992.

Table 2. Top 5 Percent and 1 Percent: Data vs. Model

	Earning Share		Income Share		Wealth Share	
	Top 5%	Top 1%	Top 5%	Top 1%	Top 5%	Top 1%
Data	30%	14%	26%	12%	54%	29%
Model	30%	11%	28%	11%	48%	18%

Note: The data for income, earnings, and wealth are obtained from the SCF 1992.

Labor supply elasticity We estimate the labor supply elasticity implied by the model in the spirit of Rogerson and Wallenius (2009). To do so, we generate artificial panel data from our model economy with 5000 households for 100 quarters and aggregate it to the annual level. We run the following regression:

$$\ln h_{i,t} = b_0 + b_w \ln \widetilde{w}_{i,t} + b_c \ln c_{i,t} + \varepsilon_{h,t} \qquad \text{with} \qquad \varepsilon_{h,t} \overset{iid}{\sim} \mathcal{N}(0, {\sigma_h}^2),$$

where $\widetilde{w}_{i,t} = w_t z_{i,t} g(h_{i,t})/h_{i,t}$. b_w is the micro Frisch elasticity of labor supply. The average estimate of b_w is 0.4, which falls within the range of the empirical estimates (MaCurdy, 1981). In Table 3, we report the point estimate of b_w for three productivity and wealth groups (tertiles). The table reveals that the higher the productivity, the lower the labor supply elasticity, consistent with Ma (2022*a*). Moreover, it shows that the higher the wealth, the higher the labor supply elasticity.

Table 3. Labor Supply Elasticity

	Low	Middle	High
Productivity	0.59	0.35	0.17
Wealth	0.08	0.24	0.81

Note: The model-implied labor supply elasticity by productivity and wealth, which is obtained by running the following regression: $\ln h_{i,t} = b_0 + b_w \ln \widetilde{w}_{i,t} + b_c \ln c_{i,t} + \varepsilon_{h,t}$, where b_w is the labor supply elasticity.

3.2 Aggregate Dynamics

In this subsection, we show that our calibrated model matches the business cycle facts well. Table 4 compares the second moments implied by the model and the data. Consumption, C, is non-durable consumption expenditures and services; investment, I, is the sum of gross private domestic investment and durable expenditures; and output, Y, is the GDP in the private sector. The data source for consumption, investment, and output is the St. Louis Fed's FRED II database. Aggregate hours worked, H, and employment, E, are retrieved from the Labor Productivity and Costs (LPC) of the nonfarm business sector from the Bureau of Labor Statistics (BLS), where aggregate hours worked are the product of employment and hours per worker, H/E. Data moments for the variables above are based on HP-filtered series with smoothing parameter 1,600. In addition, we compare the cyclical properties of income inequality between the model and data. Our measure of income inequality is the Gini index of income, using the income data from the Current Population Survey (CPS). We compute the moments for the Gini index based on the annual frequency for both the data and model by applying an HP filter with smoothing parameter 100. The sample period of all series is 1967Q1-2020Q4.

In the table, we report the (relative) volatilities of key variables and their cross-correlations with output. The model produces the second moments in the data reasonably well. The volatility of output in the model and data is exactly the same due to our calibrated standard deviation of aggregate productivity shocks. Consumption is about half as volatile as output, and investment is about three times as volatile as output both in the model and data. Moreover, the volatility of aggregate hours in the data more than doubles that in the model. The higher volatility in the data than in the model is also observed for both extensive and intensive margins of labor supply, i.e., employment and hours per worker. Thus, even with an extensive margin and rich heterogeneity, the model has a limitation in replicating the facts on hours worked, consistent with Chang et al. (2019). Lastly, our model is quite successful in replicating the cyclicality of income inequality, which is mea-

Table 4. Business Cycle Statistics

	$\sigma(Y)$	$\sigma(C)/\sigma(Y)$	$\sigma(I)/\sigma(Y)$	$\sigma(H)/\sigma(Y)$	$\sigma(E)/\sigma(Y)$	$\sigma(H/E)/\sigma(Y)$
Data	2.14	0.55	3.11	0.79	0.72	0.19
Model	2.14	0.38	3.24	0.30	0.20	0.10
	$\rho(C,Y)$	$\rho(I,Y)$	$\rho(H,Y)$	$\rho(E,Y)$	$\rho(H/E,Y)$	$\rho(Gini,Y)$
Data	0.77	0.92	0.88	0.81	0.57	-0.50
Model	0.92	0.98	0.97	0.97	0.93	-0.80

Note: $\sigma(x)$ and $\rho(x,Y)$ are the standard deviation of variable x and the cross-correlation of x with output Y, respectively. C, I, H, E, and $Gini$ denote consumption, investment, total hours, employment, and the income Gini coefficient, respectively. The Gini coefficients in the model are annualized to be consistent with the data. All variables are logged and then detrended using the HP filter.

sured by the Gini coefficient. The income inequality is countercyclical both in the model and data, with that in the model being more countercyclical.[13] The countercyclical nature of income inequality in the model mainly arises from changes in the extensive margin of labor supply of low productivity households, as documented in Castaneda, Díaz-Giménez and Ríos-Rull (1998) and Kwark and Ma (2021).

4 Welfare Effects of Business Cycles

In this section, we use our empirically-realistic model to study the main theme of the paper: the welfare effect of business cycles. Our primary interest is in studying how the welfare effect of business cycles differs across heterogeneous households. We first compute the total welfare effect for an individual household and then decompose the total welfare effect into two parts: level and volatility effects.

4.1 Heterogeneous Welfare Effects

What are the welfare consequences of aggregate uncertainty for individual households? This subsection answers this question, which is the main focus of this paper. We compute the welfare effect of business cycles for an individual household by comparing its unconditional value function and its value function in the steady state.[14] Let $V(a, \beta, z; \overline{A}, \overline{\mu})$ be the steady-state value function for an individual with capital a, discount factor β, and productivity z, given aggregate states $\overline{A}$ and $\overline{\mu}$, where $\overline{A}$ and $\overline{\mu}$ are the steady-state aggregate

[13] Kwark and Ma (2021) find the lagging behavior of the income distribution, so we follow them and report the cross-correlation between current output and the two-year lagged Gini coefficient.

[14] The steady state in our model is defined as the equilibrium with idiosyncratic shocks but without aggregate shocks, following Aiyagari (1994).

productivity and type distribution, respectively. Let $\mathbb{E}[V(a,\beta,z;A,\mu)]$ be its unconditional value function under aggregate uncertainty, where expectations are taken over aggregate states A and μ. We obtain $\mathbb{E}[V(a,\beta,z;A,\mu)]$ by simulating the economy for 100,000 periods and computing the long-run average welfare (or the value function) for each household type.

In our model, as in Krusell and Smith (1998), all aggregate variables—consumption, the capital stock, aggregate labor services, wage, and the rate of return to capital—can be almost perfectly described as a function of two simple statistics: the mean of the wealth distribution or aggregate capital and the aggregate productivity shock. Accordingly, the aggregate state μ in the value function can be replaced by aggregate capital, K. The welfare effect of business cycles for the individual household can now be expressed as the consumption-equivalent welfare effect, λ^T, which satisfies:

$$\mathbb{E}[V(a,\beta,z;A,K)] = V(a,\beta,z;\overline{A},\overline{K},\lambda^T), \tag{4}$$

where

$$V(a,\beta,z;\overline{A},\overline{K},\lambda^T) = \max_{c_t,h_t} \mathbb{E}_0 \sum_{t=0}^{\infty} B_t \left\{ \ln(1-\lambda^T)c_t - \chi \frac{h_t^{1+1/\nu}}{1+1/\nu} \right\},$$

subject to the budget constraint (1) and the steady state factor prices, $\overline{w}$ and $\overline{r}$.

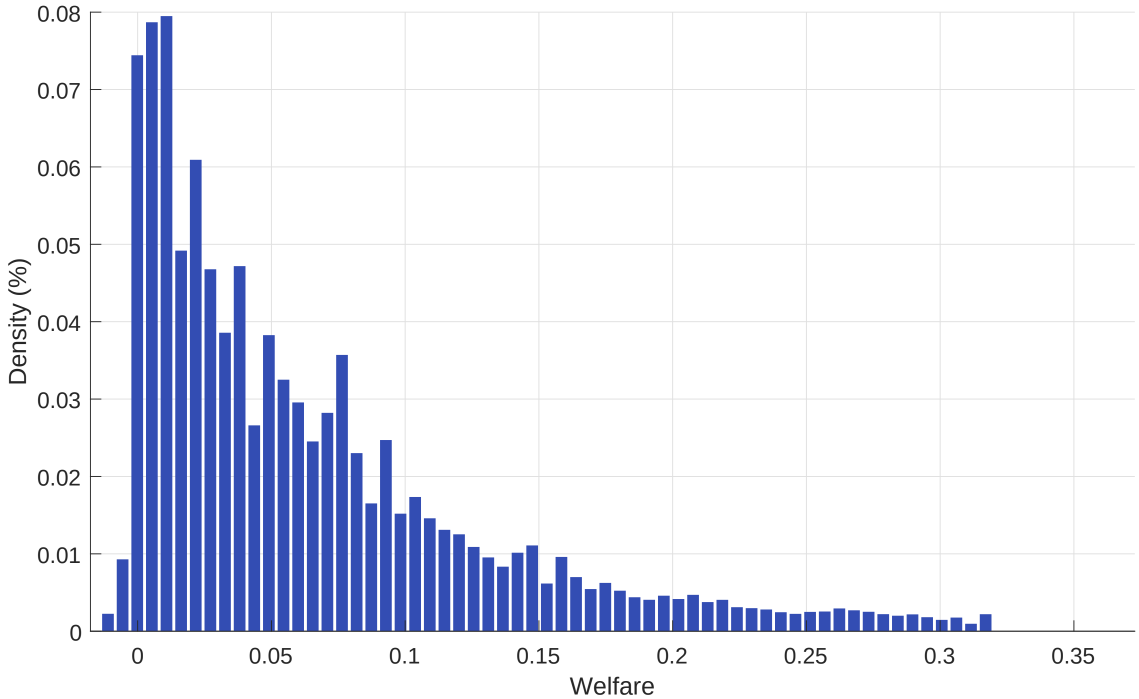
Notice that λ^T is a function of individual state variables, i.e., $\lambda^T = \lambda^T(a,\beta,z)$. Positive λ^T means that the household prefers higher aggregate volatility, while its negative value means that the household prefers the steady state, i.e., lower aggregate volatility.

Since the model features rich heterogeneity across individual households, one can expect that the consumption-equivalent welfare effect, λ^T, is widely dispersed. Figure 2 shows the distribution of households over the consumption-equivalent welfare effects, λ^T.[15] As shown in the figure, the welfare effects of business cycles are heterogeneous across households. The distribution is concentrated at the mean value of 0.0674, with the minimum and maximum being -0.0111 and 0.3662, respectively.

Interestingly, the welfare effects are positive for most individuals, indicating that most households benefit from business cycle fluctuations that stem from aggregate productivity shocks. Using representative-agent business cycle models, Cho, Cooley and Kim (2015) and Lester, Pries and Sims (2014) find that the welfare gain from aggregate uncertainty is positive over a plausible range of preference parameters. We identify that their result

[15] The density is computed based on the steady state distribution, $\overline{\mu}$.

Figure 2. Welfare Effects of Business Cycles



Note: The x-axis denotes λ^T that solves (4). The numbers are multiplied by 100 and are interpreted as a percentage of consumption.

holds for most households in our unequal economy.

Who benefits the most from business cycles? To answer this question, we report the welfare effect in greater detail in Table 5, which shows the average welfare gains conditional on individual characteristics. Observing the table, we see that patient households (β_H) benefit more than impatient households (β_L) on average. Moreover, one can see that the higher the labor productivity, the higher the welfare gains.[16] Lastly, the welfare effects increase with the level of wealth.[17]

In our economy, the discount factor and productivity are highly correlated with the level of wealth: households with high discount factors and labor productivity tend to have a high level of wealth. Specifically, the average net worth for patient households is twelve times greater than that of impatient households. Moreover, households in the highest productivity group hold around nine times the average assets of the households in the lowest productivity group. Given the positive correlation of the wealth level with the discount fac-

[16]We construct 11 grid points of labor productivity for households, but to save space, we classify households into four productivity groups. The first three grid points (z_1, z_2, and z_3) belong to the lowest group, the next two sets of three grid points belong to the low and high groups, and the last two grid points (z_{10} and z_{11}) are the highest group.

[17]Households who belong to the first three wealth deciles borrow in the steady state, and we report the first three deciles together as a single group.

Table 5. Welfare Effects of Business Cycles by Types

Discount Factor								Average
β_H	β_L			Labor Productivity: Lowest	Low	High	Highest	
0.1092	0.0256			0.0257	0.0469	0.0897	0.1740	0.0674
Wealth Decile: 1st-3rd	4th	5th	6th	7th	8th	9th	10th	
0.0130	0.0291	0.0407	0.0559	0.0733	0.0936	0.1296	0.2155	

Note: Consumption-equivalent welfare gains of business cycles, using the welfare measure defined in (4), by households type: time discount factor β, labor productivity z, and wealth a. The numbers are multiplied by 100 and are interpreted as a percentage of consumption.

tor and productivity, we can conclude that wealth-rich households tend to enjoy business cycles more than the wealth-poor in our model.

In the next subsection, we decompose our welfare measure into two effects. By doing this, we can provide deeper insight into why most households prefer business cycles to steady state and why the welfare effects are especially high for the wealth-rich but low for the poor.

4.2 Decomposition: Volatility and Level Effects

As noted from equation (4), the welfare effects of business cycles for an individual household depend on the behavior of aggregate capital and aggregate productivity. These two statistics affect welfare both directly and indirectly. First, holding aggregate capital fixed to its steady-state level, there are two channels through which volatile aggregate productivity directly affects households. It translates into volatile household income and thus consumption. Given risk-averse households, consumption volatility is welfare-reducing, as documented in Lucas (1987). Moreover, volatile aggregate productivity allows households to reallocate labor and savings intertemporally to take advantage of periods of high aggregate productivity. For example, periods of high aggregate productivity are accompanied by high average wage and rental rate of capital. Households can enjoy higher average income than in the steady state by working and saving more in periods of high aggregate productivity and less in periods of low aggregate productivity. This intertemporal factor reallocation increases households' average income, thus improving their welfare. We label these two direct welfare effects of volatile aggregate productivity as the *volatility effect*.

Second, volatile aggregate productivity indirectly affects welfare by raising the long-run average capital stock. In particular, households self-insure against aggregate risk through precautionary savings, which lead to a higher long-run aggregate capital than in the steady state. This increased long-run average capital lowers the marginal product of capital, reducing average returns on savings.[18] However, it increases the marginal product of labor, increasing average wages. Accordingly, whether the long-run average income of a household increases or not depends on its income composition between labor and capital. We label this indirect welfare effect of volatile aggregate productivity as the *level effect*.[19]

In this subsection, we decompose the overall welfare effect into the volatility and level effects. We also investigate which effect explains the pattern of overall welfare distribution that we discussed in the previous subsection by focusing on the relative importance of the asset and labor channels. Formally, our strategy for identifying these two effects closely follows Lester, Pries and Sims (2014). The volatility effect associated with business cycles for each individual household is defined as the value for λ^V that solves:

$$\mathbb{E}[V(a,\beta,z;A',K')|\overline{A},\overline{K}] = V(a,\beta,z;\overline{A},\overline{K},\lambda^V). \tag{5}$$

where $V(a,\beta,z;\overline{A},\overline{K},\lambda^V)$ is similarly defined as in equation (4).

To understand why the conditional compensating variation, λ^V, can be interpreted as the volatility effect, it is important to note that the expectation is conditioned on steady state aggregate capital on the left-hand side of (5). This means that the welfare of the steady state economy and that of the fluctuating economy are compared, conditional on equal aggregate capital. In this case, the long-run average capital between the fluctuating and steady-state economies is almost identical.[20] Hence, the conditional compensating variation, λ^V, effectively represents the direct welfare effect of volatile aggregate productivity, holding aggregate capital fixed to its steady state, i.e., the volatility effect.

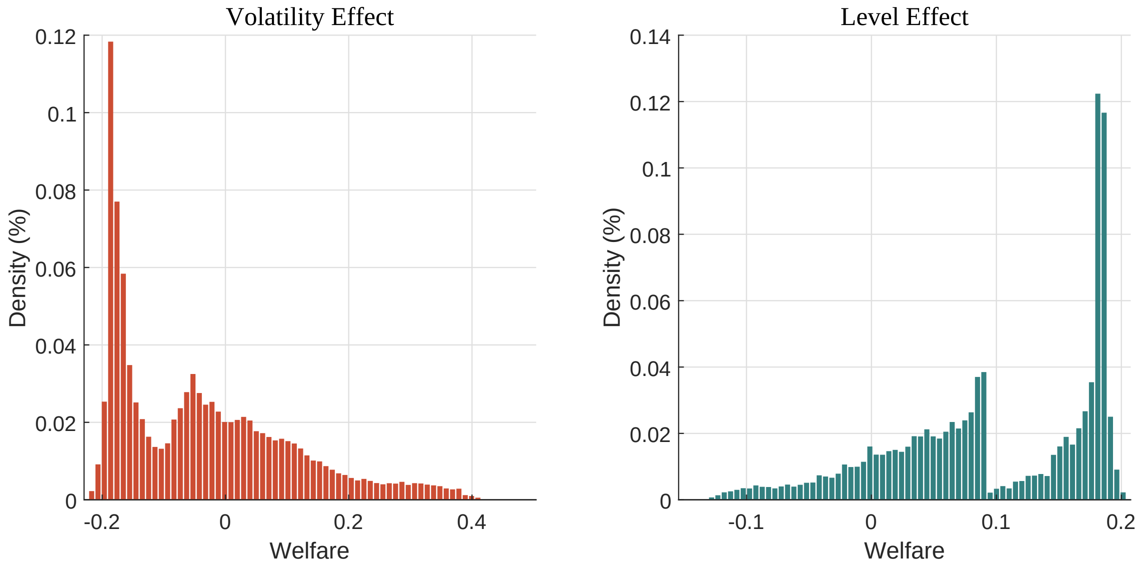
In (4), the expectation operator is the unconditional one, so the welfare of the steady-state economy and that of the fluctuating economy are compared on a different level of aggregate capital. That is, the welfare of the fluctuating economy measured in (4) takes into account the effect of high long-run average capital that stems from precautionary motives

[18] Of course, the long-run average effective labor increases with a degree of uncertainty as well, but its increase is very small compared to that of capital.

[19] The level and volatility effects defined in this paper are very similar to the mean and fluctuation effects defined in Cho, Cooley and Kim (2015) and Heiberger and Maussner (2020).

[20] We have numerically verified that the difference between long-run average capital in the fluctuating economy, $\mathbb{E}[K'|\overline{K}]$, and the steady state aggregate capital, $\overline{K}$, is very small.

Figure 3. Distribution of Volatility and Level Effects



Note: The x-axis of the left panel denotes λ^V that solves (5). The x-axis of the right panel denotes $\lambda^L (= \lambda^T - \lambda^V)$, where λ^T solves (4). The numbers are multiplied by 100 and are interpreted as a percentage of consumption.

against aggregate risk in addition to the direct welfare effect of aggregate productivity. Accordingly, the unconditional compensating variation, λ^T, captures both the volatility and level effects of business cycles. The level effect, λ^L, is then measured by the difference between the two compensating variations, i.e., $\lambda^L = \lambda^T - \lambda^V$.

Figure 3 shows the distribution of the volatility and level effects. There is substantial heterogeneity in these two effects across households, but the shapes of the two distributions are mirror images of each other. Interestingly, the distribution of the volatility effect is skewed to the right, with the minimum being -0.2120, whereas that of the level effect is skewed to the left, with the maximum being 0.2039. These contrasting shapes of the two distributions reveal that the volatility effect could be inversely related to the level effect.

To understand who benefits and loses from the volatility and level effects, we show these two effects by household type in Table 6. The total effect is simply the sum of these two effects. The table reveals an increasing pattern of volatility effects and a decreasing pattern of level effects as the discount factor, labor productivity, and wealth level increase. The pattern of volatility effects illustrates why the total welfare effects are high for wealth-rich households.

The pattern of volatility effects can be explained by two elements: 1) the shape of the consumption decision rules and 2) the difference in the households' ability to reallocate labor and savings intertemporally. Starting with the shape of the consumption decision

Table 6. Decomposition of Volatility and Level Effects by Types

	Discount Factor			Labor Productivity			
	β_H	β_L		Lowest	Low	High	Highest
Total	0.1092	0.0256	Total	0.0257	0.0469	0.0897	0.1740
Volatility	0.0801	-0.1428	Volatility	-0.0909	-0.0585	0.0005	0.1014
Level	0.0291	0.1684	Level	0.1166	0.1054	0.0892	0.0726

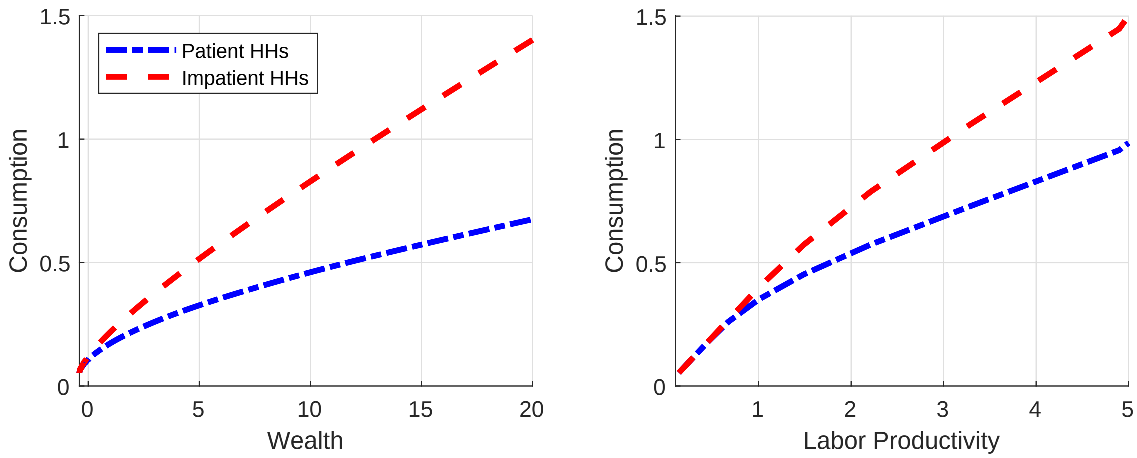
	Wealth Decile							
	1st-3rd	4th	5th	6th	7th	8th	9th	10th
Total	0.0130	0.0291	0.0407	0.0559	0.0733	0.0936	0.1296	0.2155
Volatility	-0.1617	-0.1190	-0.0864	-0.0442	-0.0038	0.0393	0.1248	0.2671
Level	0.1747	0.1481	0.1271	0.1001	0.0771	0.0543	0.0048	-0.0516

Note: The volatility effect is measured by λ^V that solves (5). The level effect is measured by $\lambda^T - \lambda^V$, where λ^T solves (4). The total effect is the sum of the volatility and level effects, which is λ^T. The numbers are multiplied by 100 and are interpreted as a percentage of consumption.

rules, Figure 4 depicts the steady state consumption decision rules for different discount factors, labor productivity, and wealth. The left panel shows the consumption decision rules across wealth levels conditional on the average productivity, while the right panel shows the consumption decision rules across productivity levels conditional on the mean asset level. The curvature of consumption decision rules becomes higher as the labor productivity, wealth level, and discount factor decrease. Households with high consumption curvature tend to experience more volatile consumption fluctuations in response to an income change, so they are the ones that substantially bear the cost of these fluctuations and show a significantly low volatility effect. Accordingly, the volatility effect increases with the discount factor, productivity, and wealth as the consumption curvature decreases with these individual characteristics.

The heterogeneity in the ability to reallocate labor and savings is another important element that explains the increasing volatility effects by the discount factor, labor productivity, and wealth level. Households can increase savings when the expected returns are high and decrease when the returns are low. Such an intertemporal adjustment of savings leads to a positive volatility effect, because it raises the individual mean capital income. We label this channel as *the asset channel of the volatility effect*. As mentioned previously, because the households with a high discount factor, labor productivity, and wealth level accumulate a relatively large amount of savings, the asset channel is the most pronounced for these households. Regarding the intertemporal adjustment of labor, households with elastic labor supply can work more when real wages are high and work less when real wages are

Figure 4. Consumption Decision Rules



Note: The left panel shows the consumption decision rules across wealth levels conditional on the average productivity, while the right panel shows the consumption decision rules across productivity levels conditional on the mean asset level.

low. This intertemporal labor adjustment contributes to a positive volatility effect because it raises the individual mean labor income. This is *the labor channel of the volatility effect*.[21]

In our model, the dominance of the asset channel over the labor channel explains the increasing volatility effect with the discount factor, labor productivity, and wealth level. To understand the dominance of the asset channel, we consider the productivity dimension as an example. As documented in Ma (2022*b*), the labor supply elasticity is higher for less productive households in a heterogeneous-agent model with a non-convex labor supply like ours. Accordingly, if the labor channel dominates the asset channel, the volatility effect should decrease in productivity. However, as shown in Table 6, it follows that the volatility effect is the smallest for households in the lowest productivity group, indicating that the asset channel plays a much more crucial role than the labor channel in accounting for the volatility effect.

Regarding the level effect, its pattern can be attributed to the heterogeneity in households' income composition. As mentioned, the increased long-run average capital lowers the average real return on capital but increases the average real wages. The increase in the average real wages raises the individual mean labor income, contributing to a positive level effect. We label this as *the labor channel of the level effect*. On the other hand, the decrease in average real return on capital lowers the individual mean capital income, decreasing the

[21]If the changes in employment at the extensive margin were to be accounted for by variations in labor demand (e.g., wage rigidity models), the labor channel of the volatility effect would likely imply welfare costs of business cycles rather than gains. For instance, if firms sharply cut hiring during bad times, households would be subject to more volatile consumption than in an environment in which variations in labor demand are weak. Highly volatile consumption can reduce the welfare effect of business cycles.

level effect. This is what we label as *the asset channel of the level effect*. Households with a low discount factor, labor productivity, and wealth level have a relatively high share of labor income in their total income, so the effect of increased average wage more than offsets the effect of decreased asset returns, making them enjoy higher average income than in the steady state. In short, the labor channel dominates the asset channel for these households. However, households with a high discount factor, labor productivity, and wealth level have a relatively high share of capital income in their total income. Hence, the decreased capital income due to the reduced asset returns works to offset the increased labor income. As observed from households at the 10th wealth decile, such an offsetting force is substantially high, leading to a negative level effect. This implies that the unfavorable asset channel dominates the favorable labor channel for these households.

In summary, volatility and level effects are inversely related in the wealth dimension: the former increases with wealth, whereas the latter decreases with wealth.[22] Moreover, the increasing pattern of the total welfare effects by wealth mostly arises from the volatility effects.

4.3 Aggregation

In the previous subsections, our focus was on how the welfare effects of business cycles differ across households. A natural question is whether household heterogeneity matters for the aggregate welfare effects. To answer this question, we construct a representative agent (RA) model that shares the same preferences with those of our heterogeneous agent (HA) model in aggregate and then compare the aggregate welfare effects in the two models. The household's preference in a standard representative-agent model is given by:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left[\ln C_t - \chi \frac{H_t^{1+1/\nu}}{1+1/\nu} \right],$$

with a budget constraint:

$$C_t + \tilde{a}_{t+1} = w_t H_t + (1+r_t)\tilde{a}_t, \tag{6}$$

where $\tilde{\beta}$ is the representative household's discount factor. C_t, H_t, and $\tilde{a}_t$ are representative household's consumption, hours worked, and asset holdings. Notice that the curvature parameter over hours worked of a representative agent, ν, does not have the same meaning

[22] In Appendix B.2, we show that this relationship holds within each discount factor type.

Table 7. Welfare Effects of Business Cycles: HA vs. RA Models

	Total	Volatility	Level
HA Model	0.0674	-0.0313	0.0987
RA Model	0.0069	-0.0309	0.0378

Note: HA (RA) denotes the heterogeneous-agent (representative-agent) model. The numbers are multiplied by 100 and are interpreted as a percentage of consumption.

as that in our heterogeneous-agent model. The curvature parameter is closely related to the intensive margin elasticity in the HA model (Chang et al., 2019; Ma, 2022*a*), whereas that in the RA model corresponds to the elasticity of aggregate hours. Therefore, for the sake of fair comparison, we need to choose the value of ν for the RA model to match the elasticity of aggregate hours in the HA model. The elasticity of aggregate hours is 1.2 in the HA model, so the value of ν is 1.2.[23] We solve the RA model globally and then compare the aggregate welfare effect of business cycles between the HA and RA models.

Table 7 presents the total welfare effect and its components in the two models. Focusing on the RA model, the total welfare effect is 0.007 percent of consumption, indicating that a representative household prefers aggregate fluctuations over the deterministic steady state. This result stems from the favorable effect of the increased average capital, as reflected in the positive level effect, and is consistent with findings by Cho, Cooley and Kim (2015) and Lester, Pries and Sims (2014). Because the share of labor income in the representative agent's income is 2/3, the increased average capital is largely passed to the increased labor income, increasing the average consumption. The gain of the increased average consumption (i.e., the level effect) is greater than the cost of volatile consumption (i.e., the volatility effect), thus delivering a positive total welfare effect.

Comparing the total welfare effect and its components between the two models, we find that these effects differ between the two models, indicating that household heterogeneity does matter for the aggregate welfare effects. Given the high marginal propensity to consume (MPC) in the HA model, one might conjecture that the HA model would predict more volatile consumption than its RA counterpart and thus would lead to a lower welfare effect due to the more negative volatility effect.

However, we find that the welfare effect in the HA model is larger than that of the RA

[23]The remaining parameters in the RA model are chosen as follows. χ is chosen so that the steady-state hours worked are the same as the HA model. The representative household's discount factor is set to match the steady-state annual return on capital of 4 percent, which is the targeted moment for our baseline HA model. The capital income share and the parameters of the technology shock process are identical to those in the HA model.

model because of the greater level effect. Notice that the precautionary savings motives against the aggregate productivity shocks are present in both HA and RA models, as both models are solved non-linearly. Thus, the increase in wages due to the increased average capital under aggregate fluctuations arises in both models. However, the increase in the average capital is more pronounced in the HA model than in the RA model, since households in incomplete asset markets have stronger precautionary savings motives. Indeed, given the same aggregate shock process, the long-run average capital is 0.7 percent greater than its steady-state counterpart in the HA model, while it rises by only 0.4 percent in the RA model. Such a substantial increase in the average capital increases the average income of households with a low discount factor, low labor productivity, and low wealth through an increase in wages. As the MPC of these households is much larger than that of the representative household, the increased wages lead to greater average consumption in the HA model than in the RA model. The strong level effect due to the interaction of aggregate capital and high MPCs is an important element that makes the average level effect in the HA model greater than in the RA model.

5 Role of Model Ingredients

In this section, we study how the welfare effects vary in different model economies to understand the role of each ingredient in our baseline model. We consider four counterfactual models: the model with inelastic labor, the model with divisible labor, the model without borrowing, and the single discount factor model. As for the calibration of the model with inelastic labor, the model with divisible labor, and the model without borrowing, we choose β_H to match the target real interest rate of 4 percent and choose β_L to keep the ratio of β_H to β_L identical to the one in the baseline model. Regarding the calibration of the discount factor in the single discount factor model, we choose the discount factor to match the average capital return of 4 percent. The remaining parameters are unchanged from the baseline. It is worth noting that the steady-state quantity and distribution in the counterfactual model are different from those in the baseline models, because the discount factors have changed to maintain the real interest rate target. Thus, in Table 8, we compare the baseline model to the counterfactual models for different asset levels, which are normalized by output.

Table 8. Total and Volatility Effects across Models

		Asset-Output Ratio						
		0.1	0.5	1	5	10	50	Avg.
Baseline	Total	0.0311	0.0363	0.0429	0.0712	0.0947	0.2027	0.0674
	Volatility	-0.1117	-0.0923	-0.0875	-0.0024	0.0464	0.2610	-0.0313
Inelastic Labor	Total	0.1342	0.1273	0.1225	0.0904	0.0603	0.0019	0.1014
	Volatility	-0.1338	-0.1121	-0.0976	-0.0214	0.0413	0.1576	-0.0581
Divisible Labor	Total	0.0861	0.0809	0.081	0.0838	0.0849	0.1138	0.0869
	Volatility	-0.0250	-0.0082	-0.0020	0.0296	0.0484	0.1158	0.0148
No Borrowing	Total	0.0519	0.0575	0.0606	0.0731	0.0862	0.1419	0.0743
	Volatility	-0.0214	-0.0078	-0.0055	0.0170	0.0342	0.1025	0.0154
Single Discount	Total	0.1153	0.1160	0.1131	0.1044	0.0952	0.0710	0.1034
	Volatility	-0.0033	0.0029	0.0072	0.0320	0.0525	0.1063	0.0318

Note: The total effect is the sum of the volatility and level effects. The numbers are multiplied by 100 and are interpreted as a percentage of consumption.

Inelastic labor The model of inelastic labor assumes the time devoted to work, $h_{i,t}$, is equal to 1. The table reveals that the inelastic labor model predicts more negative volatility effects for all asset positions than the baseline model. Because of inelastic labor, labor supply cannot be used to improve consumption smoothing. As a result, households are exposed to more volatile consumption over the business cycle in the model with inelastic labor and thus show lower volatility effects for all asset levels. Interestingly, despite the increasing volatility effects by wealth, the total effects decrease with wealth in the case of inelastic labor. The reason is the strong adverse asset channel for the wealth-rich group, which significantly reduces the level effect for this group. Specifically, in the model with inelastic labor, precautionary savings are much larger than in the baseline model due to the absence of a labor adjustment margin. Such large precautionary savings induce a much lower average capital return than in the baseline model, which reduces the capital income for the wealth-rich group. This asset channel significantly lowers the level effect for wealthy individuals and thus contributes to the decreasing total effects by wealth.

Divisible labor The model of divisible labor assumes that the fixed cost of devoting time to work, Δ, is 0. Without the fixed cost, households with low wealth can utilize labor flexibly to smooth consumption, thereby achieving less volatile consumption over the business cycle than in the model with the fixed cost. Accordingly, Table 8 reveals that the model of divisible labor exhibits larger volatility effects for low-wealth households than the baseline model, which contribute to larger total effects for these households. However, for very wealthy households, volatility effects are lower in the model with divisible labor than in the baseline model. This is because of the reduced volatility in the capital return, with its coefficient of variation being 0.1239 in the baseline model and 0.1139 in the model with divisible labor. Because of the absence of the extensive margin of labor supply, the model with divisible labor produces less volatile hours worked than the baseline model. This leads to a lower variation in the marginal product capital, which is the capital return. The less volatile capital returns make good times less attractive to wealth-rich households, thereby inducing less increased average capital income than in the baseline model. Accordingly, the weak asset channel explains the low volatility and total effects for very wealthy households in the model with divisible labor compared to the baseline model.

No borrowing Table 8 shows that the model with no borrowing predicts less dispersed volatility effects along the asset dimension than the baseline model. That is, households with low wealth exhibit less negative volatility effects in the model without borrowing. Intuitively, in the baseline model, households with low wealth are likely to hit the borrowing limit during bad times as they borrow to smooth consumption. Being at the negative wealth limit, they are exposed not only to fluctuations in wage income but also to fluctuations in interest payment. However, in the model with no borrowing, they are exposed to fluctuations in wage income only. Therefore, consumption of low-wealth households is less volatile in the model without borrowing than in the baseline model, implying larger volatility effects.

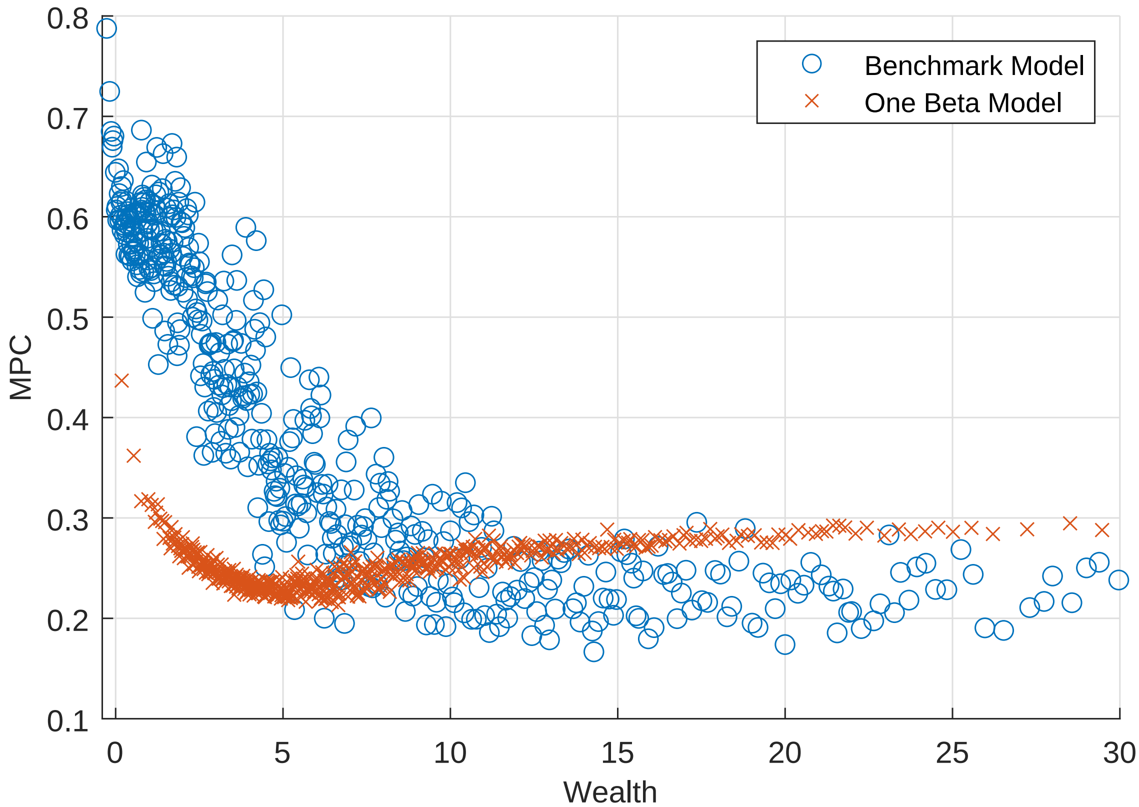
Meanwhile, households with high wealth show lower volatility effects in the model with no borrowing than in the baseline model. This is because of the less volatile capital return in the model without borrowing. In particular, the coefficient of variation for the capital return in the baseline model is 0.1239, while it is 0.1229 in the model without borrowing. As households are subject to less volatile consumption at the borrowing limit in the model without borrowing, households have a smaller incentive to stay away from the borrowing limit through precautionary savings over the business cycle. This leads to the less volatile capital stock, which generates a lower coefficient of variation for the capital return in the model with no borrowing. With less volatile capital returns, wealth-rich households are less willing to adjust their savings across time in a way that can increase the average capital income. Accordingly, the asset channel is weaker in the model with no borrowing, leading to smaller volatility effects for the wealth-rich group than in the baseline model. The less dispersed volatility effect is the dominant factor that explains the less dispersed total welfare effect in the model with no borrowing.

Single discount factor As is well known in the recent literature, the main purpose of having multiple discount factors is to produce a realistic average MPC, which is normally hard to produce with one-asset models of a single discount factor. Recent studies, such as Krueger, Mitman and Perri (2016), Hagedorn, Manovskii and Mitman (2019), Aguiar, Bils and Boar (2020), and Gornemann, Kuester and Nakajima (2021), show that models with discount factor heterogeneity can generate a realistic average MPC without relying on a two-asset model with different liquidity in the style of Kaplan, Moll and Violante (2018). Our baseline model produces a plausible average MPC thanks to the two different discount factors. The annual average MPC in our model is 0.39, which is close to 0.37, recently estimated by Gross, Notowidigdo and Wang (2020). The quarterly average MPC in our model is 0.12. This number is close to 0.16, which is the one predicted from the two-asset model of Kaplan, Moll and Violante (2018).

Thus, by comparing the single discount factor model with the baseline model, one can understand the importance of matching wealth distribution and average MPCs in shaping the distribution of welfare effects. With a single discount factor, the model produces a wealth Gini smaller than that in the data and in the baseline model (0.69 vs. 0.79). The quarterly and annual average MPCs in this single discount factor model are 0.07 and 0.25, respectively. These numbers are much lower than the quarterly and annual average MPC in the baseline model. Figure 5 shows the scatter plot of individual MPCs generated from the baseline and single discount factor model. The figure reveals that the single discount factor model exhibits a flatter MPC distribution than the baseline model. Specifically, MPCs for the low-wealth group are much smaller than in the baseline model, while MPCs for the high-wealth group tend to be slightly higher.

Such a less dispersed MPC distribution is a key factor that explains the flatter distribution of the volatility effects in the single discount factor model. Table 8 reveals that, compared to the baseline model, the volatility effects in the single discount factor model are larger for the low-wealth group but smaller for the high-wealth group. Thanks to the smaller MPCs, wealth-poor households in the single discount factor model are subject to less volatile consumption fluctuations than in the baseline model, which is a feature that explains the larger volatility effects in the single discount factor model. In addition, the larger

Figure 5. MPCs of Individual Households



Note: Individual MPCs by wealth in the baseline model (Benchmark Model) and those in the single discount factor model (One Beta Model).

MPCs for wealth-rich households in the single discount factor mean that these households save less of their income than in the baseline model. Accordingly, wealth-rich households in the single discount factor model are less inclined to concentrate their savings in periods of high capital return, reaping a lower average capital income than in the baseline model. Such a weaker asset channel for the wealth-rich group explains a smaller volatility effect in the single discount factor model than in the baseline model for this group.

Another difference between the two models is the total welfare effects. Although the volatility effect increases with wealth in both models, the total effect decreases with wealth in the single discount factor model but not in the baseline model. This difference comes from the less dispersed volatility effects across wealth levels in the single discount factor model. With relatively flat volatility effects, the total effects in the single discount factor model are mostly driven by level effects, which increase with wealth.

Relationship to Krusell et al. (2009) As noted in the introduction, the present paper is closely related to the work of Krusell et al. (2009), who uncover the welfare effects of business cycles in a heterogeneous agent business cycle model. Importantly, Krusell et al. (2009) find welfare losses from business cycles, whereas we find welfare gains. Having investigated the role of each model ingredient in shaping the welfare effect, we are ready to discuss the features of our model that can account for these differences.[24] The main reasons for these differences come from the level effect and the elasticity of labor supply.

As explained, the level effect captures the effect of the increased long-run average capital that arises from precautionary savings against aggregate risk. The level effect is an important component that increases the welfare effects of business cycles by increasing the household average wages (Lester, Pries and Sims, 2014; Cho, Cooley and Kim, 2015). Compared to us, Krusell et al. (2009) underestimate the level effect because of the way they compute the welfare of the fluctuating economy. In particular, they compute the conditional welfare, which conditions on a randomly drawn capital stock. This capital stock is not the one that stems from precautionary savings against aggregate risk, so their level effect is underestimated compared to ours. If we rule out the level effect and consider conditional welfare as the welfare metric, the welfare effect would capture only the volatility effect. In this case, we obtain a negative welfare effect, as in Krusell et al. (2009), implying welfare losses from business cycles.

However, even if we only consider the volatility effect as the welfare metric, our average welfare cost is smaller than theirs: 0.03% of consumption in our model, while 0.08% in theirs. This difference can be partly explained by the assumption of the labor supply. Specifically, the labor supply is elastic in our economy, but inelastic in Krusell et al.'s (2009). With inelastic labor, wealth-poor households can no longer reduce consumption volatility by adjusting the labor margin. Thus, households are subject to more volatile consumption on average in the economy of Krusell et al. (2009), which contributes to a higher average welfare cost. To get a sense of the extent to which the labor margin is important in reducing consumption volatility, we compare the average volatility effects between the baseline economy and the counterfactual economy in which labor supply is inelastic in Table 8. The table shows that the volatility effect is -0.03 in the baseline model, but -0.06 in the model of inelastic labor supply. The difference in these numbers indicates that the absence of elastic labor leads to a greater cost of consumption fluctuations.

[24] Although the predecessor of Krusell et al. (2009) is Krusell and Smith (1999), Krusell et al. (2009) mentioned that they computed the welfare cost of business cycles inadvertently and incorrectly in Krusell and Smith (1999). Thus, we compare our results with those of Krusell et al. (2009).

6 Conclusion

This paper studies the welfare effects of business cycles from a disaggregate perspective in a heterogeneous-agent model, which incorporates market incompleteness and non-convexity into the mapping from the time devoted to work to labor services. We find that the welfare effects of business cycle fluctuations are positive, and the effects are significantly heterogeneous across households. Particularly, wealth-rich households tend to enjoy business cycles the most. We then decompose the total welfare effect into volatility and level effects to identify why the wealth-rich benefit the most and find that the two effects are inversely correlated in the wealth dimension: the volatility effect increases with wealth, whereas the level effect decreases with wealth. Hence, the volatility effect largely accounts for the increasing pattern of the total welfare effects by wealth. This indicates that wealth-rich households benefit the most from aggregate fluctuations because they can smooth consumption to a large extent and can enjoy higher average income through reallocating savings intertemporally.

Future research could incorporate multiple types of assets with different degrees of liquidity (as in Kaplan, Moll and Violante, 2018), alternative aggregate shock, including demand and investment-specific shocks (as in Lester, Pries and Sims, 2014), nominal rigidity, and the open economy features to study how these ingredients change the welfare effects of business cycles found in the present paper.

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Appendix

A The Computational Algorithm

A.1 Stationary Economy

We describe the computational algorithm used to solve for the steady-state equilibrium. To do so, we construct $N_a = 151$ grids for asset holdings, a, in the range of $[\underline{a}, 200]$, with more grid points assigned on the smaller values of a. We also construct $N_z = 11$ equally spaced grids for logged individual labor productivity, $\hat{z} = \ln z$, in the range of $[-3\sigma_z/\sqrt{1-\rho_z^2}, 3\sigma_z/\sqrt{1-\rho_z^2}]$. The transition probability matrix for individual labor productivity, π_z, is obtained using the Tauchen (1986) method. Given these grid points and the transition matrix, we find the stationary measure, $\overline{\mu}$, following four steps:

Step 1. Have guesses for endogenous parameters such as χ, Δ, $\pi_z(z_{11}|z_{11})$, σ_z, β_H, and β_L.

Step 2. Solve for the optimal decision rules for saving and hours worked, $\{a'(a,\beta,z), h(a,\beta,z)\}$, and the value function, $V(a,\beta,z)$, by iterating on the Bellman equation:

(a) Compute the steady-state real wage using the firm's first-order condition, given the steady-state capital return, $\overline{r}$, of 4 percent.

(b) Guess the value function, $V_0(a,\beta,z)$, given the grids for a and z.

(c) Solve the optimization problem for each employment status:

$$V_1^E(a,\beta,z) = \max_{a' \geq \underline{a},\, h \geq \underline{h}} \Big\{ \ln\left(\overline{w}g(h)z + (1+\overline{r})a - a'\right) - \chi\frac{h^{1+1/\nu}}{1+1/\nu} + \beta\sum_{z'}\sum_{\beta'}\pi_z(z'|z)\pi_\beta(\beta'|\beta)V_0(a',\beta',z') \Big\},$$

and

$$V_1^U(a,\beta,z) = \max_{a' \geq \underline{a}} \left\{ \ln\left((1+\overline{r})a - a'\right) + \beta\sum_{z'}\sum_{\beta'}\pi_z(z'|z)\pi_\beta(\beta'|\beta)V_0(a',\beta',z') \right\}.$$

(d) Compute $V_1(a,\beta,z)$ as $V_1(a,\beta,z) = \max\left\{V_1^E(a,\beta,z), V_1^U(a,\beta,z)\right\}$.

(e) If V_0 and V_1 are close enough for all grid points, go to the next step. Otherwise, update the value functions ($V_0 = V_1$), and go back to (c).

Table A.1. ESTIMATES AND ACCURACY OF FORECASTING RULES

Dependent Variable	Coefficient Cons.	$\ln K$	$\ln A$	R^2	Den Haan (2010) Error Mean (%)	Max (%)
$\log K'$	0.11956	0.94272	0.10639	0.9999	0.1636	0.4133
$\log w$	-0.16084	0.41419	0.83378	0.9999	0.1195	0.4077

Step 3. Using the household's decision rules obtained from Step 2 and the transition probability matrices for z and β, compute the time-invariant measure, $\overline{\mu}$.

Step 4. Compute the targeted moments using $\overline{\mu}$. If the targeted moments are sufficiently close to the assumed ones, then the steady-state equilibrium of the economy is attained. Otherwise, reset the endogenous parameters, and go back to Step 1.

A.2 Economy with Aggregate Shocks

To solve the heterogeneous-agent economy with aggregate shocks, one needs to track the distribution of households, μ, because this infinite-dimensional object affects prices and thus the household's decision. We use the Krusell and Smith (1998) method, which approximates the distribution with the first moment and requires a log-linear forecasting function to approximate the evolution of the distribution and the market-clearing wage. The method involves the following steps:

Step 1. We construct grids for aggregate state variables, such as TFP shocks and aggregate capital, and individual state variables, such as individual labor productivity and asset holdings. Specifically, we construct equally spaced 5 grid points for aggregate capital, K, and equally spaced 11 grid points for TFP shocks. The grids for individual state variables are the same as those constructed for solving the steady-state equilibrium.

Step 2. Guess the coefficients for the log-linear forecasting functions for K' and w.

Step 3. Given the guessed forecasting functions for K' and w, we solve the optimization problems for households and obtain the policy functions for asset holdings, $a'(a, \beta, z; K, A)$, consumption $c(a, \beta, z; K, A)$, and hours worked, $h(a, \beta, z; K, A)$.[25]

[25] Given the wage rate, w, the return on capital, r, can be obtained from the firm's profit maximization. As in the steady-state economy, the transition probabilities for z and A are constructed using the Tauchen (1986) method.

Step 4. We simulate the model for 3,500 periods using the policy functions obtained in Step 3 following the procedure below:

(a) Set the initial conditions for K, A, and $\mu(a, \beta, z)$.

(b) Using the hours decision rule, $h(a, \beta, z)$, check if the labor supply equals the labor demand, i.e., $\int zg(h(a, \beta, z))d\mu = N^D$.[26] If not, update w so that it clears the labor market.

(c) Given the market-clearing wage, solve the optimization problem for individual households to update the policy functions for asset holdings, $a'(a, \beta, z)$, consumption $c(a, \beta, z)$, and hours worked, $h(a, \beta, z)$.

(d) Obtain aggregate variables using the type distribution, μ, where $C = \int c(a, \beta, z)d\mu$, $N = \int zg(h(a, \beta, z))d\mu$, $K' = \int a'(a, \beta, z)d\mu$, $H = \int h(a, \beta, z)d\mu$, $Y = AK^\alpha N^{1-\alpha}$, and $I = K' - (1 - \delta)K$.

(e) Obtain the next period measure $\mu'(a, \beta, z)$ using $a'(a, \beta, z)$ and the transition probability matrices for z and β. Repeat (b)-(e) until we have a time series with a sample size of 3,500.

Step 5. We obtain the new coefficients for the forecasting functions by the OLS estimation using the simulated time series.[27] If the new coefficients are close enough to the previous ones, move to the next step. Otherwise, we update the coefficients and go to Step 3.

Step 6. Once the coefficients are converged, we compute the unconditional welfare by simulating the economy for 100,000 periods using a series of aggregate shocks and taking the average of the welfare (or value function) over time for each household type.

Table A.1 summarizes the estimated coefficients, the goodness of fit, and the accuracy of the forecasting rules. It is clear that the R^2s for all forecasting functions are very large. We also check the accuracy of forecasting rules based on the statistics proposed by Den Haan (2010). We find that the mean Den Haan (2010) errors are sufficiently small (not exceeding 0.17 percent), and the maximum errors are also reasonably small (less than 0.5 percent) for all forecasting functions.

[26] Given the wage rate, labor demand, N^D, can be obtained from the firm's first-order condition.

[27] We drop the first 500 periods to eliminate the impact of the arbitrary choice of initial aggregate state variables.

Table A.2. Idiosyncratic Shocks and Distribution

	Consumption	Income	Earnings	Wealth
Data	0.33	0.51	0.61	0.79
Baseline	0.38	0.56	0.61	0.79
No Disc. Factor	0.37	0.60	0.65	0.69
No Productivity	0.03	0.08	0.05	0.59

Note: Gini coefficients in the data and models. The data for income, earnings, and wealth are obtained from the SCF 1992.

B Additional Figures and Tables

B.1 Idiosyncratic Productivity and Discount Factor Shocks

We investigate the importance of idiosyncratic productivity shocks and discount factor shocks in matching the distribution at the steady state. To do so, we consider two alternative models: One shuts down the discount factor heterogeneity, and the other one shuts down the idiosyncratic productivity shocks.

Regarding the model without discount factor heterogeneity, we keep all the other parameters fixed to the baseline calibration and adjust the common discount factor to match the annual return on capital of 4 percent. By comparing the distributions in the baseline model and the model without discount factor heterogeneity, one can understand the role of the discount factor shocks in shaping the distributions. Table A.2 shows that, when discount factor shocks are absent, the model produces a smaller wealth Gini than the baseline model, implying a less dispersed wealth distribution. This observation indicates that discount factor heterogeneity is important in generating a dispersed wealth distribution in the data.

The table also presents the distributions in the model without idiosyncratic productivity shocks, which assumes that the standard deviation of idiosyncratic productivity is zero. In this model, β_H is chosen to match the annual return on capital of 4 percent, and β_L is chosen such that β_H/β_L is the same as that in the baseline model. The other parameters are fixed to the baseline calibration. The table reveals that, when idiosyncratic productivity shocks are absent, the model generates equal consumption, income, and earnings distribution compared to the baseline model. This result is not surprising, given that the main purpose of having idiosyncratic productivity shocks is to match the earnings Gini in the data.

Table A.3. Welfare Effects of Business Cycles across Discount Factors

	Labor Productivity			
	Lowest	Low	High	Highest
β_H	0.0499	0.0802	0.1397	0.2672
β_L	0.0014	0.0137	0.0397	0.0808

	Wealth Decile							
	1st-3rd	4th	5th	6th	7th	8th	9th	10th
β_H	0.0305	0.0434	0.0529	0.0653	0.0798	0.1007	0.1322	0.2192
β_L	0.0101	0.0221	0.0313	0.0435	0.0609	0.0737	0.0766	0.1054

Note: Consumption-equivalent welfare gains of business cycles across discount factors. The numbers are multiplied by 100 and are interpreted as a percentage of consumption.

B.2 Welfare Effects across Discount Factors

Table A.3 shows that the result that the average welfare gain is increasing with labor productivity and wealth holds within each discount factor. One interesting observation is the interaction between labor productivity and discount factors. In particular, the increase in the welfare gain by labor productivity is more pronounced for the highest discount factor group than for the lowest discount factor group. This observation reveals that the welfare gain is particularly large for those who have high earnings and a high willingness to save.

In addition, in Table 5, we showed that the volatility effects increase with the labor productivity and wealth levels, while the level effects decrease with the labor productivity and wealth levels. Table A.4 illustrates that this inverse relationship between volatility and level effects emerges within each discount factor as well. The increasing pattern of volatility effects by labor productivity and wealth level within each discount factor plays a dominant role in driving the increasing total welfare in labor productivity and wealth level.

Asset and labor channels In the body of the paper, we explained that the asset channel is more important than the labor channel in explaining the increasing volatility effects by productivity and wealth. This argument holds for each discount factor, as shown in Table A.4. That is, within each discount factor, households with high wealth and productivity have more room to save than those with low wealth and productivity, and thus they can enjoy high mean capital income by saving more when the return is high and less when the return is low. This asset channel is vital in driving the increasing volatility effects

Table A.4. Volatility and Level Effects across Discount Factors

		Labor Productivity			
		Lowest	Low	High	Highest
Total	β_H	0.0499	0.0802	0.1397	0.2672
	β_L	0.0014	0.0137	0.0397	0.0808
Volatility	β_H	0.0045	0.0444	0.1181	0.2696
	β_L	-0.1867	-0.1618	-0.1175	-0.0671
Level	β_H	0.0454	0.0358	0.0216	-0.0024
	β_L	0.1881	0.1755	0.1572	0.1479

		Wealth Decile							
		1st-3rd	4th	5th	6th	7th	8th	9th	10th
Total	β_H	0.0305	0.0434	0.0529	0.0653	0.0798	0.1007	0.1322	0.2192
	β_L	0.0101	0.0221	0.0313	0.0435	0.0609	0.0737	0.0766	0.1054
Volatility	β_H	-0.0602	-0.0428	-0.0251	-0.0008	0.0306	0.0686	0.1287	0.2713
	β_L	-0.1782	-0.1559	-0.1326	-0.0991	-0.0666	-0.0392	0.0466	0.1407
Level	β_H	0.1138	0.1013	0.0893	0.0739	0.0539	0.0322	-0.0016	-0.0700
	β_L	0.1946	0.1773	0.1605	0.1357	0.1122	0.0956	0.0246	-0.0400

Note: The total effect is the sum of the volatility and level effects. The numbers are multiplied by 100 and are interpreted as a percentage of consumption.

within each discount factor type. The asset channel is particularly pronounced for households with a high discount factor across all productivity and wealth levels, as they prefer to save more than households with a low discount factor.

The level effect is related to the increased long-run average capital, which increases wages but decreases the return on capital. In the body of the paper, we argued that the labor channel explains a large level effect for households with low productivity and wealth, while the asset channel explains a small level effect for households with high productivity and wealth. Table A.4 reveals that this is true for each discount factor. That is, within each discount factor, households with low wealth and productivity rely more on labor income than capital income for their consumption. Thus, they enjoy high average wages, as observed from the large level effect. However, because households with high wealth and productivity rely more on capital income than labor income for consumption, the asset channel is more significant than the labor channel for them. Accordingly, the level effect is very small for these households, which are harmed by the low return on capital. Notice that the level effect is larger for the low discount factor type than the high discount factor type for all productivity and wealth deciles. This is because the unfavorable asset channel matters less for the low discount factor group, as it tends to save less than the high discount factor group.