First-Price and Second-Price Auctions with Externalities: An Experimental Study*

Chulyoung Kim† Sang-Hyun Kim‡ Jinhyuk Lee§ Jaeok Park¶

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Abstract

We consider a scenario where a single indivisible object is auctioned off to three bidders and among the three bidders there is one bidder whose winning imposes a positive or negative externality on the other two bidders. We theoretically and experimentally compare two standard sealed-bid auction formats, first-price and second-price auctions, under complete information. Using a refinement of undominated Nash equilibria, we analyze equilibrium bids and outcomes in the two auction formats. Our experimental results show that overbidding relative to equilibrium bids is prevalent, especially in second-price auctions, and this leads to higher revenue and lower efficiency in second-price auctions than in first-price auctions, especially under negative externalities. Our results are consistent with previous experimental findings that bidders tend to overbid more in second-price auctions than in first-price auctions, and they suggest that such a tendency is robust to the introduction of externalities.

Keywords: auctions; externalities; experiments; overbidding; efficiency.

JEL: C91; D44; D62.

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†School of Economics, Yonsei University, Seoul, Korea (chulyoung.kim@gmail.com).

‡School of Economics, Yonsei University, Seoul, Korea (kimsan46@gmail.com).

§Corresponding author, Department of Economics, Korea University, Seoul, Korea (jinhyuklee@korea.ac.kr).

¶School of Economics, Yonsei University, Seoul, Korea (jaeok.park@yonsei.ac.kr).

1 Introduction

An auction is a widely used mechanism to allocate items or resources, and bidders participating in an auction may experience externalities due to post-auction interactions. For example, if a telecommunications company obtains a frequency band in a spectrum auction, it can offer better services to its customers, and its rival companies may suffer from reduced market shares. While it is natural to have negative externalities among competing firms, a winning bidder may create positive externalities on others. For instance, if a person wins a painting at an auction and displays it in his house, a close friend of his who often visits his house will benefit from his winning. Our examples suggest that externalities occurring among auction participants can be positive or negative, and in addition that they can be identity-dependent in the sense that some bidders (e.g., those having stronger rivalry or friendship) impose or incur greater externalities than others do.

In this paper, we theoretically and experimentally investigate two standard sealed-bid auction formats, namely, first-price and second-price auctions, in a setting where there are positive or negative identity-dependent externalities among bidders. Although more complicated sealed-bid auctions may perform better in the presence of externalities, we focus on these two auction formats because they are widely used in practice and have simple rules that participants in an experiment can easily understand. In addition, we consider the following relatively simple setting in order to facilitate our theoretical and experimental investigation. There are three bidders who participate in an auction of a single indivisible object. One of the three bidders is called Red and the other two Blue. The Red bidder's winning the object creates the same externality on the two Blue bidders, while a Blue bidder's winning imposes no externality on the other bidders. Though simple, this setting allows us to capture interesting features of externalities in that we can deal with identity-dependent externalities that are positive or negative.

Our main focus in our theoretical and experimental study is on the complete information scenario where the three bidders' valuations of the object and the externality exerted by the

Red bidder are common knowledge among the bidders.[1] Although it is a usual practice to assume incomplete information in the study of auctions, studying auctions with complete information can provide useful insights on bidders' strategic motives, especially in complicated auction environments. Moreover, the assumption of complete information is relevant in situations where a group of bidders participates in the same kind of auctions (for example, spectrum auctions and procurement auctions) repeatedly so that they get to know each other's types. In the existing literature on auctions with externalities, Hoppe et al. (2006) and Ettinger (2010) present theoretical analyses under complete information.[2] Also, regarding generalized second-price auctions for sponsored search advertising, Edelman et al. (2007) and Varian (2007) conduct theoretical investigation and Che et al. (2017) and Bae and Kagel (2019) perform experimental studies, considering complete information settings. In our theoretical analysis, we study noncooperative equilibria of the games induced by the two auction formats. In order to reduce the multiplicity of equilibria, we propose a refinement of undominated Nash equilibrium called *effectively undominated Nash equilibrium* and adopt it as the equilibrium concept for our analysis. We characterize effectively undominated Nash equilibria under various conditions on the three bidders' valuations and the externality.

In the benchmark case of no externalities, it is well-known that both auction formats allocate the object to the bidder with the highest valuation and achieve the revenue equal to the second highest valuation at any undominated Nash equilibrium. In the presence of externalities, we can consider two cases regarding efficient allocations: one where it is efficient for the bidder with the highest valuation to obtain the object, and the other where the presence of externalities makes it inefficient for the bidder with the highest valuation to obtain the object. The former case occurs if the Red bidder has the highest valuation under positive externalities, or if a Blue bidder has the highest valuation under negative externalities. We show that the results under no externalities can be generalized to the former case. The latter case, which is more interesting in our view, occurs if the Red bidder does not have

[1] In our experiments, we also consider an incomplete information scenario where each bidder knows only her own valuation and the externality

[2] See also Bernheim and Whinston (1986) who develop a theory of first-price package auctions under the assumption of complete information.

the highest valuation but it is efficient for her to obtain the object accounting for the positive externality, or if the Red bidder has the highest valuation but it is inefficient for her to obtain the object accounting for the negative externality. We show that the equilibrium allocation can be efficient or inefficient depending on the valuations and the externality in the latter case. We study the two cases for both positive and negative externalities and cover the total four cases in Propositions 1–4.

Based on our theoretical results, we make three predictions for our experiments: (1) both auction formats yield the same allocation and revenue, (2) Blue bidders' bids and the revenue decrease in the externality level when the Red bidder wins the object, and (3) inefficient allocations are more likely when there are inefficient equilibria than when there are only efficient equilibria. In order to test these predictions, we conducted laboratory experiments with the two treatments of first-price and second-price auctions. In our experiments, we used predetermined parameter sets for the valuations and the externality so that we can cover the cases of positive and negative externalities evenly and focus on the more interesting case where it is inefficient for the bidder with the highest valuation to receive the object.

Our experimental data reveal that participants tend to overbid relative to equilibrium bids, especially in second-price auctions. The result that participants overbid more in second-price auctions than in first-price auctions has been reported in the extant experimental literature (see, for example, Kagel, 1995), and various behavioral motives such as spitefulness and the joy of winning have gained attention in explaining observed overbidding behavior (see, for example, Andreoni et al., 2007; Cooper and Fang, 2008; and Kimbrough and Reiss, 2012).[3] Our experimental results suggest that bidders' tendency to overbid in second-price auctions is robust to the introduction of externalities. We find that the two auction formats achieve similar revenue and efficiency under positive externalities, consistently with Prediction 1, but not under negative externalities, where second-price auctions yield higher revenue and less efficient allocations than first-price auctions. This finding suggests that standard models have higher explanatory power for the case of positive externalities. In or-

[3] Bartling and Netzer (2016) show that cognitive skills are negatively correlated with overbidding, and Filiz-Ozbay and Ozbay (2007) show that the feeling of regret can explain overbidding behavior in first-price auctions.

der to test Prediction 2, we focus on the case of positive externalities because the Red bidder wins at equilibrium only under positive externalities in our experiments. We find that Blue bidders' bids and the revenue decrease in the externality level under positive externalities, consistently with Prediction 2, except that the predicted effect of externalities on Blue bidders' bids are not statistically significant in second-price auctions. Lastly, our experimental data contradict Prediction 3 as the existence of inefficient equilibria is shown to have no significant effect on efficiency.

Since the seminal paper by Jehiel et al. (1996), a large theoretical literature has developed on auctions with externalities. Jehiel et al. (1996) consider a situation where a potential bidder decides whether to participate in an auction and she cannot avoid externalities even if she does not participate in the auction. They construct an optimal auction in a complete information scenario as well as in an incomplete information scenario where each bidder knows only her own valuation and the externalities she imposes on the other bidders. Jehiel et al. (1999) study a multidimensional mechanism design problem in another incomplete information scenario where each bidder knows only her own valuation and the externalities the other bidders impose on her.[4] Das Varma (2002) examines a situation where each bidder's winning imposes a negative externality on exactly one other bidder, and he compares an open ascending-bid auction with standard sealed-bid auctions. Hoppe et al. (2006) study license auctions where a potential entrant's entry exerts a negative externality on incumbents. Ettinger (2010) considers a situation where bidders care about the identity of the winner and the price paid by the winner, and he compares first-price and second-price auctions under complete information. Recently, Jeong (2019) proposes multidimensional second-price and English auctions with externalities and studies their properties, while Jeong (2020) analyzes the core of a cooperative auction game with externalities.

Compared to the large theoretical literature on auctions with externalities, the experimental literature on this topic is surprisingly small. A closely related work to ours is Hu et al. (2013). They theoretically and experimentally investigate free riding in an auction where

[4] See also Jehiel and Moldovanu (1995, 1996, 2000) for studies on related models and Caillaud and Jehiel (1998) for a study on collusion in auctions with externalities.

an entrant's winning imposes a negative externality on two incumbents, and they compare an English ascending price auction and a first-price sealed-bid auction in terms of bidding behavior, revenue, and efficiency. Another related work is Goeree et al. (2013). They consider multi-unit license auctions where an entrant's entry creates a negative external effect on two incumbents, and they compare an ascending auction and a discriminatory auction, focusing on incumbents' incentives for demand reduction and preemptive bidding. As in the settings of the above two papers, we consider an auction where there are three bidders and there is one bidder whose winning imposes a negative externality on the other two bidders. While these papers focus on negative externalities, we allow both positive and negative externalities. Also, while the above two papers, as well as Das Varma (2002), compare an open auction and a sealed-bid auction, we compare two sealed-bid auction formats.

The rest of the paper is organized as follows. Section 2 presents a theoretical analysis of our setting for the two auction formats. Section 3 describes our experimental design and procedures, and Section 4 provides theoretical predictions for our experiments. Section 5 shows our main experimental results, and Section 6 concludes. Proofs of the propositions are presented in Appendix A, our experimental results on the incomplete information setting in Appendix B, and the experimental instructions in Appendix C.

2 Theoretical Analysis

There are three bidders (called bidders 1, 2, and 3) and an indivisible object. When bidder i receives the object, she obtains utility $v_i \geq 0$, for all $i = 1, 2, 3$. In addition, when bidder 1 receives the object, each of bidders 2 and 3 obtains utility $e \in \mathbb{R}$. That is, bidder 1's obtaining the object creates an externality on the other bidders, while bidder $j \neq 1$ exerts no externality on the others. In this sense, we consider externalities that depend on the identity of the imposer. We allow both positive and negative externalities, and thus there is no restriction on the sign of e. We refer to v_i as bidder i's *valuation* of the object, e as the *externality* or the *externality level*, and $|e|$ as the *magnitude of the (positive/negative) externality*. We assume that the valuations are distinct across the bidders. In our theoretical analysis, we focus on a complete information scenario in which the valuations and the externality are commonly known among the bidders.[5]

We consider two auction formats to allocate the object, first-price and second-price auctions. In each auction format, each bidder i simultaneously submits a bid $b_i \geq 0$, and the bidder who submits the highest bid wins the object. The winning bidder pays the highest bid in a first-price auction and the second highest bid in a second-price auction. In the following, we study equilibria of the games induced by the two auction formats, considering the three cases of no, positive, and negative externalities.

2.1 No Externalities

As a benchmark, we first consider the case where there are no externalities (i.e., $e = 0$). In this case, all the three bidders are symmetric in the externality structure, and we assume that $v_1 > v_2 > v_3$ without loss of generality.

Let us consider the game induced by the first-price auction format. It can be shown that a bid profile (b_1, b_2, b_3) is a Nash equilibrium if and only if $b_1 \in [v_2, v_1]$, $b_1 \geq b_j$ for all $j \neq 1$, and $b_1 = b_j$ for some $j \neq 1$, assuming that ties are broken in favor of a bidder with a lower index.[6] At any Nash equilibrium, bidder 1 obtains the object, and thus the efficient allocation of the object is achieved.[7] Since bidding more than one's own valuation is weakly dominated in a first-price auction, a bid profile (b_1, b_2, b_3) is an undominated Nash equilibrium if and only if $b_1 = b_2 = v_2$ and $b_3 \leq v_3$. Hence, bidder 1 pays the second highest valuation, v_2, at any undominated Nash equilibrium.

Let us consider the game induced by the second-price auction format. Every bidder has a weakly dominant strategy of bidding one's own valuation in a second-price auction.

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[5] As explained in the Introduction, it is not uncommon to focus on a complete information scenario if the auction game itself is complicated and if participants have a chance to learn others' valuations.

[6] Consider an alternative scenario where the bid space is discrete and ties are broken with equal probability, which is the case in our experiments. Let $\Delta > 0$ be the unit of bids, and let us assume that the valuations are multiples of Δ. In this scenario, a bid profile (b_1, b_2, b_3) such that $b_1 \in [v_2, v_1 - \Delta]$, $b_1 > b_j$ for all $j \neq 1$, and $b_1 = b_k + \Delta$ for some $k \neq 1$ is a Nash equilibrium. As Δ goes to zero, any such Nash equilibrium (b_1, b_2, b_3) approaches one with $b_1 = b_k$ and the tie broken in favor of bidder 1. With this interpretation in mind, when we look for Nash equilibria where a particular bidder obtains the object, we will break ties in favor of that bidder.

[7] In our analysis, we assume that bidders have quasilinear utility functions, and thus an efficient allocation of the object maximizes the sum of bidders' utilities including those from externalities.

Hence, the bid profile $(b_1, b_2, b_3) = (v_1, v_2, v_3)$ is the unique undominated Nash equilibrium. At the undominated Nash equilibrium, bidder 1 obtains the object and pays v_2. In addition, there are Nash equilibria where bidders use weakly dominated strategies. For example, a bid profile (b_1, b_2, b_3) such that $b_i > v_1$ and $b_j = 0$ for all $j \neq i$ is a Nash equilibrium for all $i = 1, 2, 3$, and an inefficient allocation can arise at such a Nash equilibrium.

In summary, in both auction formats, bidder 1 obtains the object and pays v_2 at any undominated Nash equilibrium. That is, if we focus on undominated Nash equilibria, both auction formats yield the efficient allocation and the revenue equal to the second highest valuation.

2.2 Positive Externalities

We next consider the case where bidder 1's obtaining the object creates a positive externality on bidders 2 and 3 (i.e., $e > 0$). Since bidders 2 and 3 are symmetric in the externality structure, we assume that $v_2 > v_3$ without loss of generality. We say that bidder j competes with bidder $k \neq j$ at the bid profile (b_1, b_2, b_3) if bidder k is the highest bidder among the bidders other than bidder j. That is, when bidder j competes with bidder k, bidder j needs to outbid bidder k in order to become the highest bidder. For all $j \neq 1$, let $\tilde{v}_j = v_j - e$, and we call $\tilde{v}_j$ bidder j's *effective valuation* against bidder 1. For any $j \neq 1$, bidder j's maximum willingness to pay for the object is given by her effective valuation if she competes with bidder 1, while it is given by her valuation if she competes with bidder $k \neq 1, j$. When the externality is positive, bidder j's effective valuation is lower than her valuation (i.e., $\tilde{v}_j < v_j$) for all $j \neq 1$. In the case of positive externalities, we refine undominated Nash equilibria as follows. First, for both auction formats, we require that $b_j \leq \tilde{v}_j$ if bidder $j \neq 1$ competes with bidder 1. Second, for a second-price auction, we also require that $b_j = v_j$ if bidder $j \neq 1$ does not compete with bidder 1. We refer to an undominated Nash equilibrium satisfying these two requirements as an *effectively undominated Nash equilibrium*. These requirements can be interpreted as eliminating bids of bidder $j \neq 1$ that cannot be justified given her correct belief about whether she competes with bidder 1 or not. In subsequent theoretical analysis, we take an effectively undominated Nash equilibrium as our equilibrium concept, and we sometimes simply refer to it as an equilibrium.

We use different valuations and externality levels in our experiments (as listed in Table 1 in Section 3), and we can classify those used in the case of positive externalities into two cases.

Case P1. $v_1 > v_2$

In this case, bidder 1 has the highest valuation, and it is efficient for bidder 1 to obtain the object. In the following proposition, we study the allocation and the revenue at equilibrium in the two auction formats in this case.

Proposition 1. *Suppose that $v_1 > v_2 > v_3$ and $e > 0$. In both auction formats, bidder 1 obtains the object and pays $\tilde{v}_2$ at any effectively undominated Nash equilibrium.*

All the proofs of the propositions in this section are presented in Appendix A, and they describe equilibrium bid profiles. In a second-price auction, there are inefficient equilibria, as in the case of no externalities. However, if we focus on effectively undominated Nash equilibria, both auction formats achieve the efficient allocation and the revenue equal to the higher of the losing bidders' effective valuations. As the magnitude of the positive externality becomes larger, bidders 2 and 3 bid less aggressively against bidder 1, and thus the revenue decreases. As the magnitude approaches zero, equilibrium bids converge to those in the case of no externalities.

Case P2. $v_1 + 2e > v_2 > v_1$

In this case, bidder 1 does not have the highest valuation, but the magnitude of the positive externality is large enough to make it efficient for bidder 1 to obtain the object.

Proposition 2. *Suppose that $v_1 + 2e > v_2 > v_1$, $v_2 > v_3$, and $e > 0$. In both auction formats, the following statements hold.*

(i) There is an effectively undominated Nash equilibrium where bidder 1 obtains the object if and only if $v_1 \geq \tilde{v}_2$, and she pays $\tilde{v}_2$ at any such equilibrium.

(ii) There is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $\tilde{v}_2 \geq v_1 > v_3$ or $v_3 > v_1$, and she pays $\max\{v_1, v_3\}$ *at any such equilibrium.*

(iii) There is no effectively undominated Nash equilibrium where bidder 3 obtains the object.

When the magnitude of the positive externality is large enough to have $v_1 \geq \tilde{v}_2$, the efficient allocation is achieved at equilibrium in both auction formats. On the contrary, when the magnitude is not so large or bidder 1 has the lowest valuation, an inefficient allocation can arise at equilibrium. If $v_3 > v_1 \geq \tilde{v}_2$, there are both efficient and inefficient equilibria. In this case, competition between bidders 2 and 3 may result in the inefficient equilibrium where bidder 2 obtains the object, but they prefer enjoying the positive externality from bidder 1's winning at the efficient equilibrium.[8]

2.3 Negative Externalities

Lastly, we consider the case where bidder 1's obtaining the object creates a negative externality on bidders 2 and 3 (i.e., $e < 0$). Again, we assume that $v_2 > v_3$ without loss of generality. When the externality is negative, bidder j's effective valuation against bidder 1 is higher than her valuation (i.e., $\tilde{v}_j > v_j$) for all $j \neq 1$. Hence, in the concept of effectively undominated Nash equilibria for the case of negative externalities, we require that $b_j \leq v_j$ if bidder $j \neq 1$ does not compete with bidder 1, and for a second-price auction, we also require that $b_j = \tilde{v}_j$ if bidder $j \neq 1$ competes with bidder 1. We classify the parameter sets with negative externalities used in our experiments into two cases.

Case N1. $v_2 > v_1$

In this case, bidder 2 has the highest valuation, and it is efficient for bidder 2 to obtain the object.

Proposition 3. *Suppose that $v_2 > v_1$, $v_2 > v_3$, and $e < 0$. In both auction formats, bidder 2 obtains the object and pays* $\max\{v_1, v_3\}$ *at any effectively undominated Nash equilibrium.*

[8] Note that $v_2 - \max\{v_1, v_3\} \leq v_2 - v_1 < 2e$ in Case P2. Thus, the total payoff of bidders 2 and 3 is higher when bidder 1 receives the object than when bidder 2 does at the price $\max\{v_1, v_3\}$.

When there are no externalities, the bidder with the highest valuation wins the object at any undominated Nash equilibrium. The presence of a negative externality increases bidder 2's maximum willingness to pay for the object when she competes with bidder 1, while it makes no difference when she competes with bidder 3. As a result, the efficient allocation is achieved at equilibrium even when the externality is negative.

Case N2. $v_1 > v_2 > v_1 + 2e$

In this case, bidder 1 has the highest valuation, but the magnitude of the negative externality is large enough to make it not efficient for bidder 1 to obtain the object.

Proposition 4. *Suppose that $v_1 > v_2 > v_1 + 2e$, $v_2 > v_3$, and $e < 0$. In both auction formats, the following statements hold.*

(i) There is an effectively undominated Nash equilibrium where bidder 1 obtains the object if and only if $v_1 \geq \tilde{v}_2$, and she pays $\tilde{v}_2$ at any such equilibrium.

(ii) There is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $\tilde{v}_2 \geq v_1$, and she pays v_1 at any such equilibrium.

(iii) There is an effectively undominated Nash equilibrium where bidder 3 obtains the object if and only if $\tilde{v}_3 \geq v_1$, and she pays v_1 at any such equilibrium.

When the magnitude of the negative externality is not so large that $v_1 \geq \tilde{v}_2$ holds, bidder 1, who has the highest valuation, obtains the object at equilibrium, as in the case of no externalities. Since $v_2 - v_1 > 2e$ and bidder 1's equilibrium bid does not exceed her valuation in both auction formats, bidders 2 and 3 can improve their total payoff by having bidder 2 outbid bidder 1. However, when $v_1 \geq \tilde{v}_2$, bidder 2 cannot gain by becoming the highest bidder, unless she receives a compensation from bidder 3. The inability of bidders 2 and 3 to behave collectively in our noncooperative equilibrium concept results in an inefficient allocation in this case.[9] When the magnitude of the negative externality is large enough to have $\tilde{v}_j \geq v_1$ for some bidder $j \neq 1$, bidder j is willing to pay more than her valuation in order to avoid the negative externality resulting from bidder 1's winning the object. When

[9] Jeong (2020) studies the core of an auction game with externalities and transferable utility, where side payments between bidders are possible.

$\tilde{v}_2 \geq v_1 > \tilde{v}_3$, only bidder 2 is willing to be the highest bidder. On the other hand, when $\tilde{v}_3 \geq v_1$, both bidders 2 and 3 can become the highest bidder at equilibrium. If bidder $j \neq 1$ is the highest bidder at equilibrium, her payoff is $v_j - v_1 < 0$ while the other bidder's payoff is zero. If both bidders 2 and 3 choose low bids in the hope that the other bidder becomes the winner, bidder 1 may become the winner.

3 Experimental Design and Procedures

We ran experimental sessions at the Center for Research in Experimental and Theoretical Economics (CREATE) managed by the School of Economics at Yonsei University in South Korea. We experimentally implemented two treatments, corresponding to the two auction formats, with a between-subject design.

In each round, participants were randomly matched into groups of three and given 170 experimental coins each. They were told that they participate in an auction for an item with their group members and that the winner obtains v coins, where v represents each bidder's valuation of the item (corresponding to v_i in Section 2) and can be different across members. In each group, one member is called "Red" and the other two "Blue." Between the two Blue participants, we call the one who has the higher valuation "Blue-High" and the other "Blue-Low." The Red participant's winning the item creates a positive or negative externality on the Blue participants: if the Red participant wins, she obtains v coins and the Blue participants obtain e coins, where e can be positive or negative. Note that a Red participant corresponds to bidder 1, a Blue-High participant to bidder 2, and a Blue-Low participant to bidder 3 in Section 2.

The rules for bidding and payments were different between the two treatments. In both treatments, each participant made a bid within her budget. That is, a bid was made as an integer between 0 and 170 coins. In a first-price auction, the highest bidder won the auction and paid her bid. In a second-price auction, the highest bidder won the auction and paid the second highest bid. Participants were informed that if there are multiple highest bidders, one bidder is randomly chosen among them by the server computer with equal chances.

These two auction formats are well known to economists as well as to laypeople.

Participants played this auction game for 25 rounds with feedback about the winner and the bids of the three participants in their groups at the end of each round. The values of valuations and externalities were predetermined for ease of comparison. Note that the parameter space for each group is vast, consisting of four numbers one of which can be positive or negative. If we had chosen parameters randomly for each group, the realized parameter sets might have covered different parameter ranges (as classified in Section 2) unevenly between the two treatments, which would have impeded comparison between the treatments. Alternatively, we could have used the same randomly generated parameter set for all the groups in each round. In this case, with less samples, it might have happened that most of the realized parameter sets cover uninteresting cases (e.g., very small magnitudes of the externality). Given the budget and time constraints, we thus chose to use predetermined parameters in order to enhance comparison between the treatments and focus on more interesting cases.

In Table 1, we list the predetermined parameters, the valuations for the three participants in each group and the externality levels, that were used in our experiments. In the first 15 rounds, we adopted a random matching protocol, forming groups in each round, and participants played an auction game with all relevant information, i.e., they knew every member's valuation and the externality. Among the 15 rounds, the first 5 rounds were practice rounds, and the next 10 rounds are called the complete information rounds. For instance, consider the parameters for Round 8 in Table 1. In Round 8, the Red participant's valuation is 33, the two Blue participants' valuations are 70 and 51, and the externality is 42.

After Round 15, new groups of three participants were randomly formed, and participants played auction games with the same members (i.e., a fixed matching protocol was used) for the final 10 rounds. In these last 10 rounds, participants played auction games with a limited amount of information: each participant knew her own valuation and the externality but not the others' valuations. We call these rounds the incomplete information

Table 1: Parameters for Valuations and Externalities

	Round	Valuations Red	Blue-High	Blue-Low	Externality
Practice	1	72	91	58	−36
	2	56	40	37	−57
	3	64	69	59	0
	4	34	52	37	25
	5	95	88	78	4
CI	6	78	95	85	35
	7	98	94	90	−64
	8	33	70	51	42
	9	94	79	67	−18
	10	81	72	63	−42
	11	42	68	41	52
	12	93	70	59	−28
	13	96	91	90	2
	14	56	63	53	9
	15	79	37	35	−48
II	Positive	46	65	63	38
	Negative	71	64	58	−29

Note: CI = complete information; II = incomplete information.

rounds. In these rounds, participants may infer the other members' valuations from feedback about their bids. Consider the last row in Table 1. If a group is assigned this set of parameters for Rounds 16–25, the Red participant's valuation is 71, the two Blue participants' valuations are 64 and 58, and the externality is −29. In this case, for example, the Red participant knows that her valuation is 71 and the externality is −29, but she does not know the two Blue participants' valuations. Groups of participants were randomly assigned to the two cases of positive and negative externalities. By implementing the incomplete information rounds in our experiments, we seek to understand the effects of private information and to check whether our findings in the complete information rounds are robust to the lack of information about the other members' valuations.

Table 2 shows information about sessions. We ran three sessions for each of the first-price auction (FPA) and second-price auction (SPA) treatments in September 2019. One of the authors led all the sessions to minimize confounding factors. In total, we invited 42 and

Table 2: Information about Treatments

Treatment	FPA	SPA
Sessions	3	3
# of participants	42	48
Average payments	21,468	20,817

Note: Payments are expressed in KRW.

Table 3: Classification of the Parameter Sets and the Summary of Equilibrium Outcomes

Case	Prop.	Bids in FPA			Bids in SPA			Win.	Pay.	Eff.	Rounds
		R	BH	BL	R	BH	BL				
P1	1	$\tilde{v}_2$	$\tilde{v}_2$	$\tilde{v}_3$	v_1	$\tilde{v}_2$	$\tilde{v}_3$	R	$\tilde{v}_2$	eff	13
P2-1	2(i)	$\tilde{v}_2$	$\tilde{v}_2$	$\tilde{v}_3$	v_1	$\tilde{v}_2$	$\tilde{v}_3$	R	$\tilde{v}_2$	eff	6, 8, 11, 14
P2-2	2(ii)	v_1	v_3	v_3	v_1	v_2	v_3	BH	v_3	ineff	6, 8
N2-1	4(ii)	v_1	v_1	v_3	v_1	$\tilde{v}_2$	v_3	BH	v_1	eff	7, 9, 10, 12, 15
N2-2	4(iii)	v_1	v_2	v_1	v_1	v_2	$\tilde{v}_3$	BL	v_1	ineff	7, 10, 15

Note: R = Red bidder, BH = Blue-High bidder, BL = Blue-Low bidder, Win. = Winner, Pay. = Winner's Payment (or Revenue), Eff. = Efficiency, eff = efficient allocation, ineff = inefficient allocation.

48 undergraduate students to the FPA and SPA treatments, respectively, from our subject pool. The experimental instructions for the two treatments are presented in Appendix C.[10] After Round 25, the experiments ended with demographic surveys (i.e., age, gender, major, religion), and one round from Rounds 6–25 was randomly chosen by the server computer for payments to participants. Each coin in the chosen round was converted to KRW 95, and participants obtained gift certificates worth their payoffs. The average payment including show-up fees (KRW 5,000) was about KRW 21,000 (around USD 17.5). Each session for the FPA and SPA treatments took about 50 minutes.

4 Theoretical Predictions

In Table 3, we classify the parameter sets used in the complete information rounds in our experiments according to the structures of equilibria. Table 3 presents the cases to which the

[10] Because the instructions for the two treatments are identical except for one paragraph, we present them together in Appendix C.

parameter sets belong, the propositions in which these cases are studied in Section 2, and the bids of the Red bidder (R; bidder 1), the Blue-High bidder (BH; bidder 2), and the Blue-Low bidder (BL; bidder 3) in first-price and second-price auctions at effectively undominated Nash equilibria. It also shows the winner and her payment (or the revenue) at equilibrium as well as the efficiency of the equilibrium allocation. Equilibrium bid profiles are derived in the proof of the propositions presented in Appendix A. For first-price auctions, there is a range of equilibrium bids for the lowest bidder, and we take the upper limit of the range in Table 3. The equilibrium bids shown in Table 3 are derived with the assumptions of a continuous bid set and arbitrary tie-breaking, and we take these values for convenience. If we assume a discrete integer bid set and random tie-breaking as in our experiments, the second highest equilibrium bid in a first-price auction is reduced by 1, while there is no change in equilibrium bids in a second-price auction.

Among the complete information rounds, Rounds 6–15, the externality is positive in Rounds 6, 8, 11, 13, and 14, while it is negative in Rounds 7, 9, 10, 12, and 15. Among the rounds with positive externalities, Round 13 belongs to Case P1 in Section 2, which is studied in Proposition 1. All the other rounds with positive externalities belong to Case P2, which is divided into P2-1 and P2-2 in Table 3. In Case P2-1, which is studied in Proposition 2(i), there are efficient equilibria. On the other hand, in Case P2-2, which has $v_3 > v_1$ and is studied in Proposition 2(ii), there are inefficient equilibria. It is possible that a parameter set belongs to the two cases simultaneously, which is the case for Rounds 6 and 8. All the rounds with negative externalities belong to Case N2, which is divided into N2-1 and N2-2 in Table 3. Cases N2-1 and N2-2 are studied in Proposition 4(ii) and (iii), respectively. There are efficient equilibria in Case N2-1, while there are inefficient equilibria in Case N2-2. The condition for Case N2-2 implies that for Case N2-1, and thus Case N2-2 is a subcase of Case N2-1. Rounds 7, 10, and 15 belong to Case N2-2. From Table 3, it can be seen that we have chosen parameter sets with which we can compare cases where there are only efficient equilibria and those where there are both efficient and inefficient equilibria.

Based on the results in Propositions 1–4 and the classification in Table 3, we can make the following theoretical predictions for our experiments.

Prediction 1. (Comparison between the two auction formats) There is no difference between first-price and second-price auctions in terms of the allocation and the revenue.

Prediction 2. (Effects of externalities on bids and revenue) The Blue bidders' bids and the revenue decrease in the externality level when the Red bidder wins the object.

Prediction 3. (Occurrence of inefficient allocations) Inefficient allocations are more likely to occur when there are inefficient equilibria than when there are only efficient equilibria.

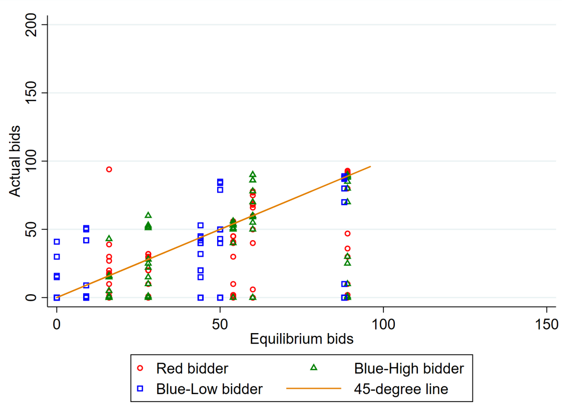
In all the cases covered in Propositions 1–4, the bidder who obtains the object and her payment are the same in the two auction formats, as long as we focus on effectively undominated Nash equilibria. Thus, we can predict that both auction formats yield the same allocation and revenue. In the cases where the Red bidder obtains the object at equilibrium (covered in Propositions 1, 2(i), and 4(i)), the equilibrium bid of a Blue bidder is given by her effective valuation and the equilibrium revenue is given by the higher of the two Blue bidders' effective valuations. Since the effective valuations decrease in the externality level e, we can expect that the Blue bidders' bids and the revenue decrease in e as well when the Red bidder wins the object. In Case P2, if $v_1 \geq \tilde{v}_2$, there is an efficient equilibrium where the Red bidder obtains the object, and if $v_3 > v_1$, there is an inefficient equilibrium as well. Hence, given that $v_1 \geq \tilde{v}_2$ holds, we can predict that inefficient allocations are more likely to occur when $v_3 > v_1$ (Rounds 6 and 8) than when $v_1 > v_3$ (Rounds 11 and 14). In Case N2, if $\tilde{v}_2 \geq v_1$, there is an efficient equilibrium where the Blue-High bidder wins the object, and if $\tilde{v}_3 \geq v_1$, there is an inefficient equilibrium as well where the Blue-Low bidder wins the object. Hence, given that $\tilde{v}_2 \geq v_1$ holds, we can expect that inefficient allocations where the Blue-Low bidder receives the object are more likely to occur when $\tilde{v}_3 \geq v_1$ (Rounds 7, 10 and 15) than when $v_1 > \tilde{v}_3$ (Rounds 9 and 12).

5 Experimental Results

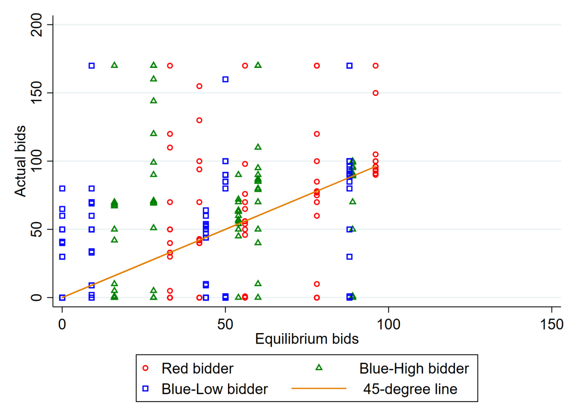
In this section, we analyze our experimental data from the complete information rounds in terms of bidding behavior, revenue, and efficiency, testing the theoretical predictions about them. We present experimental results on the incomplete information rounds in Appendix B.

5.1 Bidding Behavior

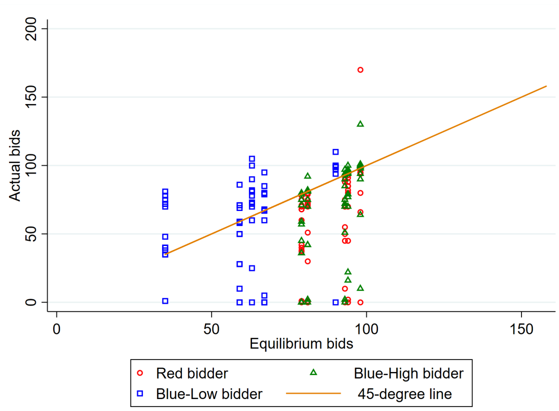
Figure 1: Bid Scatter Diagrams



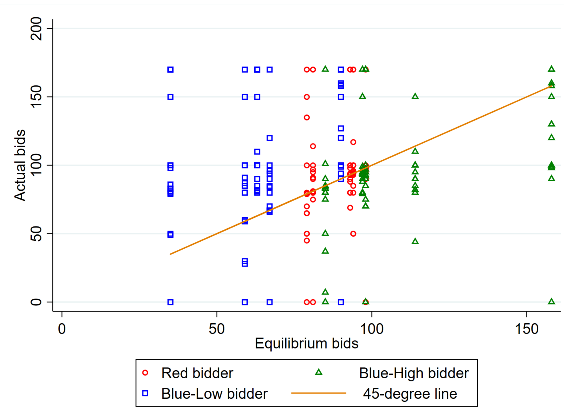
(a) FPA under Positive Externalities



(b) SPA under Positive Externalities



(c) FPA under Negative Externalities



(d) SPA under Negative Externalities

Figure 1 shows bid scatter diagrams where individual (efficient)[11] equilibrium bids are displayed on the horizontal axis and individual actual bids are displayed on the vertical axis.

[11] That is, we take efficient equilibria when there are both efficient and inefficient equilibria.

We present four bid scatter diagrams, dividing our bid data depending on the auction format and the sign of externalities. In each plot, different colors and shapes are used to distinguish bidders' roles, and the straight line represents the 45-degree line. These bid scatter diagrams are different from the standard one in that we display equilibrium bids instead of valuations on the horizontal axis. This is because the presence of externalities creates asymmetry among the participants in our experiments. With this difference, the 45-degree line makes it easy to discern whether a bidder overbid relative to her equilibrium bid.

The bid scatter diagrams suggest that participants bid more aggressively (higher) in second-price auctions than in first-price auctions regardless of the sign of externalities. This finding is consistent with the extant literature showing that overbidding is more widespread in second-price auctions than in first-price auctions. We can see in Figure 1 that in second-price auctions there are many participants who bid the maximum 170 coins or an amount close to it, regardless of their roles. Since the winner does not pay her own bid in a second-price auction, there are participants who try to win by bidding high and hope that the other members bid low. This kind of behavior is more consistent with potentially inefficient Nash equilibria where a single bidder bids high than undominated Nash equilibria.

In contrast, the winner pays her own bid in a first-price auction, and thus participants are more cautious about bidding in first-price auctions than in second-price auctions, resulting in less overbidding in first-price auctions. However, we can still observe some overbidding in first-price auctions, notably by Blue-Low bidders under negative externalities, which can be explained as follows. In our concept of effectively undominated Nash equilibria, we assume that each Blue bidder has a correct belief about the bidder with whom she competes and chooses an undominated strategy given the belief. In experiments, however, participants may have uncertainty about the opponents with whom they compete. For example, given our parameter sets, in a first-price auction where the externality is negative, the Blue-Low bidder is supposed to bid no more than her valuation at efficient equilibria, correctly believing that she competes with the Blue-High bidder. If the Blue-Low bidder mistakenly believes that she competes with the Red bidder, she is willing to bid up to her effective valuation, which is higher than her valuation.

Formally, we report Probit regression results in Table 4, where the dependant variable is the incidence of overbids. The variable takes 1 if a bidder overbids relative to her equilibrium bid and 0 otherwise. The explanatory variables are "SPA," v, e, and "Red," where "SPA" is the indicator variable for the SPA treatment, v is the bidder's own valuation, e is the externality level, and "Red" is the indicator variable that represents whether the bidder is a Red bidder or not (i.e., 1 if the bidder is Red and 0 otherwise). Columns (1) and (4) show that in the case of positive externalities, bidders overbid 24.8% points more frequently in second-price auctions than in first-price auctions, while in the case of negative externalities, they overbid 16.1% points more often. We find that overbidding is more frequent in second-price auctions, and the effect is stronger under positive externalities. This confirms our findings from the bid scatter diagrams in Figure 1. In particular, active overbidding by Blue-Low bidders in first-price auctions under negative externalities results in a weaker effect of the SPA treatment under negative externalities than under positive externalities. Columns (2) and (5) show that bidders tend to overbid more often as the magnitude of the externality increases. As can be seen in Table 3, in the case of positive externalities, a Blue bidder's (efficient) equilibrium bid is her effective valuation. As the magnitude of the positive externality increases, a Blue bidder's equilibrium bid decreases, and it makes overbidding occur more frequently. In the case of negative externalities, overbidding occurs when a Blue bidder uses her effective valuation to determine her bid. Since a Blue bidder's effective valuation increases in the magnitude of the negative externality, overbidding becomes more likely as the magnitude increases. Columns (3) and (6) show that in the case of positive externalities, Red and Blue bidders overbid at a similar rate, while in the case of negative externalities, Red bidders overbid about 24% points less frequently. This is consistent with our observation that Blue-Low bidders tend to overbid a lot, especially in first-price auctions under negative externalities.

Table 5 provides Tobit regression results where the dependent variable is overbid sizes.[12]

[12] An overbid size is defined as the difference between the bid and the equilibrium bid if the bid is higher than the equilibrium bid and zero otherwise.

Table 4: Estimation Results for the Incidence of Overbids

Variables	Incidence of Overbids					
	Positive Externalities			Negative Externalities		
	(1)	(2)	(3)	(4)	(5)	(6)
SPA	0.657***	0.665***	0.664***	0.424***	0.444***	0.478***
	(0.145)	(0.147)	(0.147)	(0.141)	(0.139)	(0.137)
v		0.003	0.003		-0.012***	-0.003
		(0.003)	(0.003)		(0.004)	(0.004)
e		0.008**	0.008**		-0.010***	-0.009***
		(0.003)	(0.003)		(0.003)	(0.003)
Red			-0.016			-0.669***
			(0.168)			(0.182)
Observations	450	450	450	450	450	450
Log-pseudo likl.	-289.8	-287.4	-287.4	-295.9	-286.9	-278.2

Note: Standard errors are clustered at the subject level. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

Table 5: Estimation Results for the Sizes of Overbids

Variables	Overbid Sizes					
	Positive Externalities			Negative Externalities		
	(1)	(2)	(3)	(4)	(5)	(6)
SPA	39.23***	39.25***	39.02***	30.94***	31.55***	33.37***
	(6.878)	(6.777)	(6.957)	(7.290)	(6.956)	(6.970)
v		0.226*	0.206*		-0.668***	-0.410***
		(0.123)	(0.115)		(0.136)	(0.148)
e		0.774***	0.763***		-0.435***	-0.390***
		(0.132)	(0.129)		(0.135)	(0.131)
Red			-2.648			-22.05***
			(7.078)			(6.742)
Observations	450	450	450	450	450	450
Log-pseudo likl.	-1,119.1	-1,106.3	-1,106.2	-1,089.1	-1,073.7	-1,068.2

Note: Standard errors are clustered at the subject level. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

Table 6: Estimation Results for Blue Bidders' Bids under Positive Externalities

Variables	Bids			
	Blue-High Bidders		Blue-Low Bidders	
	FPA	SPA	FPA	SPA
	(1)	(2)	(3)	(4)
v	0.776***	0.675*	0.580***	0.849***
	(0.194)	(0.334)	(0.169)	(0.274)
e	-0.588***	-0.052	-0.348*	-0.050
	(0.152)	(0.258)	(0.194)	(0.275)
Observations	70	80	70	80
R-squared	0.293	0.038	0.268	0.152

Note: Standard errors are clustered at the subject level. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

Columns (1) and (4) show that, when we consider overbidding bidders, they bid about 30 to 40 more coins (about 18% to 24% of their budgets) in second-price auctions than in first-price auctions. Columns (2) and (5) show that bidders overbid more as the magnitude of the externality increases. Columns (3) and (6) show that, in the case of positive externalities, Red and Blue bidders overbid at a similar size, while in the case of negative externalities, Red bidders bid 22 less coins (about 14% of their budgets) than Blue bidders. These results imply that each explanatory variable has a qualitatively similar effect on the incidence of overbids and overbid sizes.

We can summarize our findings on overbidding as follows.

Result 1. *Overbidding is prevalent in both treatments, especially in second-price auctions and by Blue bidders under negative externalities.*

Because our theoretical predictions in Section 4 are made based on equilibrium analysis, we can expect that there will be more inconsistency with the predictions in situations where overbidding is more severe. Prediction 2 in Section 4 predicts that the Blue bidders' bids decrease in the externality level when the Red bidder wins the object. As can be seen in Table 3, in our experiments, the Red bidder wins only in the case of positive externalities.

Table 6 shows that the coefficients for e are all negative but they are economically and statistically significant only in the FPA treatment in columns (1) and (3). Therefore, our data are consistent with the prediction about the Blue bidders' bids in Prediction 2 only in first-price auctions, not in second-price auctions. This result is due to the fact that bidders tend to bid very aggressively in second-price auctions regardless of the externality levels, as we have seen in Figure 1. Note that the absolute value of the coefficient for e in the FPA treatment is smaller for Blue-Low bidders than for Blue-High bidders. This can be explained by Blue-Low bidders' tendency to use their valuations instead of their effective valuations, which results in their overbidding in first-price auctions. Our results about the effect of externalities on Blue bidders' bids can be summarized as follows.

Result 2. *Under positive externalities, Blue (especially, Blue-High) bidders' bids decrease in the externality level in first-price auctions, not in second-price auctions.*

To sum up, Blue bidders' bidding behavior is closer to the equilibrium prediction in first-price auctions than in second-price auctions, because overbidding is more severe in second-price auctions.

5.2 Revenue

Table 7 provides ordinary least squares (OLS) regression results where the dependant variable is revenue. The explanatory variable "reference v" represents the valuation that appears in the expression of revenue at efficient equilibria. As can be seen in Table 3, in our experiments, the revenue at efficient equilibria is given by $\tilde{v}_2$ in the case of positive externalities and v_1 in the case of negative externalities. Thus, we set the reference v as the Blue-High bidder's effective valuation in the case of positive externalities and the Red bidder's valuation in the case of negative externalities.

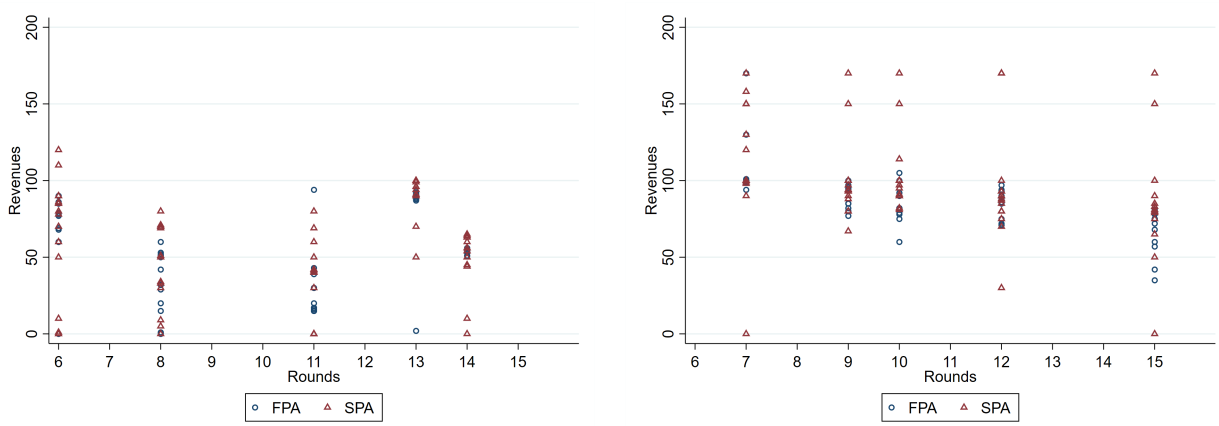
In column (1) of Table 7, we find that the two auction formats yield similar revenues under positive externalities, but column (4) reveals that second-price auctions generate higher revenue than first-price auctions under negative externalities. Hence, only the case of positive externalities is consistent with the prediction about revenue in Prediction 1. In Figure 2,

Table 7: Estimation Results for Revenue

Variables	Revenue					
	Positive Externalities			Negative Externalities		
	All	FPA	SPA	All	FPA	SPA
	(1)	(2)	(3)	(4)	(5)	(6)
SPA	2.341			11.02**		
	(3.715)			(3.928)		
reference *v*	1.000***	1.075***	0.934***	0.517***	0.621***	0.426*
	(0.161)	(0.170)	(0.264)	(0.126)	(0.101)	(0.219)
e	-0.614***	-0.644***	-0.588***	-0.144	-0.149	-0.139
	(0.0896)	(0.116)	(0.136)	(0.134)	(0.109)	(0.232)
Observations	150	70	80	150	70	80
R-squared	0.422	0.541	0.341	0.177	0.461	0.066

Note: Robust Standard errors are in parentheses. The notation *** indicates significance at 1% level, ** at 5% level, * at 10% level.

Figure 2: Revenues



(a) Positive Externalities

(b) Negative Externalities

we observe revenues equal to the maximum 170 coins or an amount close to it in several groups in second-price auctions under negative externalities, while we observe more concentrated revenues in the other cases. Overbidding in second-price auctions relative to first-price auctions occurs more frequently and severely under positive externalities, as shown in Tables 4 and 5, but its effect on the revenue is stronger under negative externalities.

From Table 7, we can also see that a larger magnitude of the externality reduces revenue under positive externalities in columns (1)–(3), but increases revenue under negative externalities in columns (4)–(6) to a lesser degree and insignificantly. Prediction 2 predicts that the revenue decreases in the magnitude of the externality in the case of positive externalities where the Red bidder wins at efficient equilibria, while we can also predict that the revenue is independent of the externality in the case of negative externalities. Thus, our results can be considered as consistent with these predictions about revenue.

Our results on revenue can be summarized as follows.

Result 3. *The two auction formats have no difference in revenue under positive externalities, but second-price auctions yield higher revenue than first-price auctions under negative externalities. An increase in the externality level reduces revenue under positive externalities and has no effect on it under negative externalities.*

5.3 Efficiency

In order to study the effect of the existence of inefficient equilibria on efficiency, we compare the cases where there exist only efficient equilibria with those where there exist inefficient equilibria as well in a similar environment. For this reason, we drop data from Round 13 in the following analysis of efficiency.

Table 8 shows the Probit regression results in which the dependant variable is the indicator variable for efficient allocations. The explanatory variable "ineff eqm" is the variable indicating the rounds having inefficient equilibria along with efficient equilibria (i.e., Rounds 6 and 8 in the case of positive externalities and Rounds 7, 10, and 15 in the case of negative externalities). Column (1) shows that the two auction formats yield similar proportions of

Table 8: Estimation Results for Efficiency

Variables	Incidence of Efficient Allocations					
	Positive Externalities			Negative Externalities		
	All	FPA	SPA	All	FPA	SPA
	(1)	(2)	(3)	(4)	(5)	(6)
SPA	0.041			-0.535**		
	(0.235)			(0.214)		
ineff eqm	-0.322	-0.186	-0.439	-0.721	-0.746	-0.694
	(0.242)	(0.356)	(0.333)	(0.443)	(0.639)	(0.616)
e	0.008	0.013	0.004	-0.015	-0.012	-0.018
	(0.008)	(0.011)	(0.010)	(0.014)	(0.020)	(0.019)
Observations	120	56	64	150	70	80
Log-pseudo likl.	-78.7	-36.3	-41.9	-93.9	-47.3	-46.4

Note: Robust standard errors are in parentheses. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

efficient allocations under positive externalities, while column (4) reveals that the SPA treatment reduces efficiency under negative externalities. Hence, our results are consistent with the prediction about the allocation in Prediction 1 only in the case of positive externalities. Our results suggest that overbidding by Blue-Low bidders under negative externalities leads to their winning more frequently in second-price auctions than in first-price auctions. This is consistent with our previous observation that overbidding has a stronger effect on revenue in second-price auctions than in first-price auctions under negative externalities. We also find no statistical evidence that the existence of inefficient equilibria and the externality level affect efficiency, contrary to Prediction 3.

Our results on efficiency can be summarized as follows.

Result 4. *The two auction formats have no difference in achieving efficient allocations under positive externalities, but first-price auctions achieve efficient allocations more often than second-price auctions under negative externalities. The existence of inefficient equilibria and the externality level have no effect on efficiency.*

6 Concluding Remarks

In this paper, we investigate an auction setting where there are three bidders and one of the bidders creates a positive or negative externalitiy on the other two bidders. We theoretically and experimentally compare the two standard sealed-bid auction formats, first-price and second-price auctions, in our setting under complete information. Using a refinement of undominated Nash equilibria, we derive equilibrium bids and outcomes in the two auction formats under various conditions on the valuations and the externality. Based on our theoretical results, we make three predictions for our experiments.

We observe that participants in our experiments tend to overbid, especially in second-price auctions. Although overbidding in second-price auctions relative to that in first-price auctions occurs more frequently and severely in rounds with positive externalities, it has stronger effects on outcomes in rounds with negative externalities. As a result, we observe higher revenue and lower efficiency in second-price auctions than in first-price auctions under negative externalities, while we find no significant differences between the two auction formats in terms of revenue and efficiency under positive externalities. That is, our experimental data are consistent with Prediction 1 only under positive externalities. These findings suggest that standard models are capable of organizing actual bidding behavior and outcomes when externalities are positive, whereas negative externalities seem to require additional elements in the model to enhance its predictive power. Introducing behavioral motives of bidders could be useful in this regard. For instance, a negative externality may affect a participant's emotion more significantly than a positive externality, as experimental studies have found that people react more strongly to losses than gains (that is, loss aversion introduced by Kahneman and Tversky, 1979). Our experimental results are consistent with Prediction 2, which is concerned with the effects of the externality level on bids and revenue, especially in first-price auctions. Lastly, in contrast to Prediction 3, we find no evidence that efficient allocations occur more frequently in rounds where there are only efficient equilibria than in rounds where there are inefficient equilibria as well.

Although participants do overbid in first-price auctions, their tendency to overbid is much stronger in second-price auctions, where we often observe very aggressive bidding behavior such as bidding the maximum amount. Hence, if our goal is to maximize the revenue, a second-price auction would be a better choice than a first-price auction. On the other hand, if our goal is to achieve efficient allocations and limit overbidding, a first-price auction would serve better. In our study, we choose to compare between first-price and second-price auctions because they have simple rules that participants can easily understand and are widely used in the real world. In these auction formats, bidders simply choose *one-dimensional* bids. However, in the presence of externalities, a bidder may have different effective valuations against other bidders, and one-dimensional bids may not be enough for bidders to convey relevant information about their preferences. Hence, more complicated auction formats that allow *multidimensional* bids (such as the ones proposed by Jehiel et al., 1999 and Jeong, 2019) may perform better, and it would be interesting to theoretically and experimentally compare one-dimensional auction mechanisms with multidimensional ones. We leave this topic for future research.

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A Proofs of Propositions

Proof of Proposition 1. (1) Consider a first-price auction. We first show that bidder $j \neq 1$ cannot obtain the object at any Nash equilibrium. Suppose to the contrary that bidder $j \neq 1$ obtains the object at a Nash equilibrium (b_1, b_2, b_3). Then $b_j \leq v_j < v_1$, and bidder 1 can gain by deviating to $b_1 \in (b_j, v_1)$. Hence, bidder 1 obtains the object at any Nash equilibrium, and a bid profile (b_1, b_2, b_3) is a Nash equilibrium if and only if $b_1 \in [\tilde{v}_2, v_1]$, $b_1 \geq b_j$ for all $j \neq 1$, and $b_1 = b_j$ for some $j \neq 1$, assuming that ties are broken in favor of bidder 1. Then bidders 2 and 3 compete with bidder 1 at any Nash equilibrium, and thus (b_1, b_2, b_3) is an effectively undominated Nash equilibrium if and only if $b_1 = b_2 = \tilde{v}_2$ and $b_3 \leq \tilde{v}_3$.

(2) Consider a second-price auction. Bidder 1 has a weakly dominant strategy $b_1 = v_1$. For bidder $j \neq 1$, $b_j < \tilde{v}_j$ and $b_j > v_j$ are weakly dominated by $b_j = \tilde{v}_j$ and $b_j = v_j$, respectively. Hence, if a bid profile (b_1, b_2, b_3) is an undominated Nash equilibrium, then $b_1 = v_1$ and $b_j \in [\tilde{v}_j, v_j]$ for all $j \neq 1$. Since bidder 1 is the highest bidder at any undominated Nash equilibrium, $(b_1, b_2, b_3) = (v_1, \tilde{v}_2, \tilde{v}_3)$ is the unique effectively undominated Nash equilibrium.

Proof of Proposition 2. (1) Consider a first-price auction. Let us assume that ties are broken in favor of bidder 1. Suppose that there is a Nash equilibrium (b_1, b_2, b_3) where bidder 1 obtains the object. In order to prevent deviations by bidders 1 and 2, we need to have $b_1 \leq v_1$ and $e \geq v_2 - b_1$, respectively, which implies $v_1 \geq \tilde{v}_2$. Suppose that $v_1 \geq \tilde{v}_2$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 1 obtains the object if and only if $b_1 = b_2 = \tilde{v}_2$ and $b_3 \leq \tilde{v}_3$.

Let us assume that ties are broken in favor of bidder 2. Suppose that there is an effectively undominated Nash equilibrium (b_1, b_2, b_3) where bidder 2 obtains the object. Then $b_2 = b_j$ for some $j \neq 2$. Suppose that $b_2 = b_1$. Then we need to have $b_2 = b_1 = v_1 \leq \tilde{v}_2$ and $v_1 > v_3 \geq b_3$. Suppose that $b_2 = b_3$. Then we need to have $b_2 = b_3 = v_3 < v_2$ and $v_3 > v_1 \geq b_1$. Thus, we obtain $\tilde{v}_2 \geq v_1 > v_3$ or $v_3 > v_1$. Suppose that $\tilde{v}_2 \geq v_1 > v_3$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $b_2 = b_1 = v_1$ and $b_3 \leq v_3$. Note that bidder 2 has no incentive to deviate because $v_2 - b_2 \geq e$. Suppose that $v_3 > v_1$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $b_2 = b_3 = v_3$ and $b_1 \leq v_1$.

Suppose that bidder 3 obtains the object at a Nash equilibrium (b_1, b_2, b_3). Then $b_3 \leq v_3$. Since $v_2 > v_3$, bidder 2 can gain by deviating to $b_2 \in (b_3, v_2)$. Hence, there is no Nash equilibrium where bidder 3 obtains the object.

(2) Consider a second-price auction. Let us assume that ties are broken in favor of bid-

der 1. At any undominated Nash equilibrium, bidder 1 chooses $b_1 = v_1$. At any effectively undominated Nash equilibrium where bidder 1 obtains the object, bidders 2 and 3 compete with bidder 1 and thus choose $b_j = \tilde{v}_j$ for all $j \neq 1$. Hence, there is an effectively undominated Nash equilibrium where bidder 1 obtains the object if and only if $v_1 \geq \tilde{v}_2$. If such an equilibrium exists, it is given by $(b_1, b_2, b_3) = (v_1, \tilde{v}_2, \tilde{v}_3)$, and bidder 1 pays the second highest bid $\tilde{v}_2$ at the equilibrium.

Let us assume that ties are broken in favor of bidder 2. At any effectively undominated Nash equilibrium where bidder 2 obtains the object and competes with bidder 1, bidder 2 chooses $b_2 = \tilde{v}_2$ and bidder 3 chooses $b_3 = v_3$. Hence, there is an effectively undominated Nash equilibrium where bidder 2 obtains the object and competes with bidder 1 if and only if $\tilde{v}_2 \geq v_1 > v_3$. If such an equilibrium exists, it is given by $(b_1, b_2, b_3) = (v_1, \tilde{v}_2, v_3)$, and bidder 2 pays the second highest bid v_1 at the equilibrium. At any effectively undominated Nash equilibrium where bidder 2 obtains the object and does not compete with bidder 1, bidders 2 and 3 choose $b_j = v_j$ for all $j \neq 1$. Hence, there is an effectively undominated Nash equilibrium where bidder 2 obtains the object and does not compete with bidder 1 if and only if $v_3 > v_1$. If such an equilibrium exists, it is given by $(b_1, b_2, b_3) = (v_1, v_2, v_3)$, and bidder 2 pays the second highest bid v_3 at the equilibrium.

Suppose that bidder 3 obtains the object at an effectively undominated Nash equilibrium (b_1, b_2, b_3). Then b_3 is either $\tilde{v}_3$ or v_3 depending on whether bidder 3 competes with bidder 1 or not. Since $v_2 > v_3 > \tilde{v}_3$, bidder 2 can gain by deviating to $b_2 > b_3$. Hence, there is no effectively undominated Nash equilibrium where bidder 3 obtains the object.

Proof of Proposition 3. (1) Consider a first-price auction. Suppose that there is a Nash equilibrium (b_1, b_2, b_3) where bidder 1 obtains the object. In order to prevent deviations by bidders 1 and 2, we need to have $b_1 \leq v_1$ and $e \geq v_2 - b_1$, respectively. Since $v_2 > v_1$ and $e < 0$, the two inequalities cannot be satisfied simultaneously, which is a contradiction. Hence, there is no Nash equilibrium where bidder 1 obtains the object.

Suppose that there is an effectively undominated Nash equilibrium (b_1, b_2, b_3) where bidder 3 obtains the object. Suppose that bidder 3 does not compete with bidder 1 at (b_1, b_2, b_3).

Then $b_3 \le v_3$. Since $v_2 > v_3$, bidder 2 can gain by deviating to $b_2 \in (b_3, v_2)$. Suppose that bidder 3 competes with bidder 1 at (b_1, b_2, b_3). Then $b_3 = b_1 \le v_1$. Since $v_2 > v_1$, bidder 2 can gain by deviating to $b_2 \in (b_3, v_2)$. In either case, we obtain a contradiction. Hence, there is no effectively undominated Nash equilibrium where bidder 3 obtains the object.

Let us assume that ties are broken in favor of bidder 2, and we look for effectively undominated Nash equilibria where bidder 2 obtains the object. Suppose that $v_1 > v_3$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $b_2 = b_1 = v_1$ and $b_3 \le v_3$. Suppose that $v_3 > v_1$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $b_2 = b_3 = v_3$ and $b_1 \le v_1$.

(2) Consider a second-price auction. Bidder 1 has a weakly dominant strategy $b_1 = v_1$. For bidder $j \neq 1$, $b_j < v_j$ and $b_j > \tilde{v}_j$ are weakly dominated by $b_j = v_j$ and $b_j = \tilde{v}_j$, respectively. Hence, if a bid profile (b_1, b_2, b_3) is an undominated Nash equilibrium, then $b_1 = v_1$ and $b_j \in [v_j, \tilde{v}_j]$ for all $j \neq 1$. We have $b_2 > b_1$ for any $b_2 \in [v_2, \tilde{v}_2]$, and thus bidder 1 cannot obtain the object at any undominated Nash equilibrium. Consider any effectively undominated Nash equilibrium (b_1, b_2, b_3) where bidders 2 and 3 compete with each other. Then we have $b_2 = v_2$ and $b_3 = v_3$. In order to have $b_1 = v_1$ as the lowest bid, we should have $v_3 > v_1$. Consider any effectively undominated Nash equilibrium (b_1, b_2, b_3) where bidder $j \neq 1$ competes with bidder 1. Then we have $b_j = \tilde{v}_j$ and $b_k = v_k$ for $k \neq 1, j$. In order to have $b_1 = v_1$ as the second highest bid, we should have $v_1 > v_3$ and $(j, k) = (2, 3)$. If $v_1 > v_3$, then $(b_1, b_2, b_3) = (v_1, \tilde{v}_2, v_3)$ is the unique effectively undominated Nash equilibrium. If $v_3 > v_1$, then $(b_1, b_2, b_3) = (v_1, v_2, v_3)$ is the unique effectively undominated Nash equilibrium.

Proof of Proposition 4. Part (i) can be proven as in the proof of Proposition 2, and we prove parts (ii) and (iii) in the following.

(1) Consider a first-price auction. Let us assume that ties are broken in favor of bidder 2. Suppose that there is a Nash equilibrium (b_1, b_2, b_3) where bidder 2 obtains the object. Then $b_2 = b_j$ for some $j \neq 2$. Suppose that $b_2 = b_3$. In order to prevent a deviation by bidder 2, we need to have $b_2 \le v_2$. Since $v_1 > v_2$, bidder 1 can gain by deviating to $b_1 \in (b_2, v_1)$, a

contradiction. Hence, it must be that $b_2 = b_1$. In order to prevent deviations by bidders 1 and 2, we need to have $b_2 \geq v_1$ and $v_2 - b_2 \geq e$, respectively. Combining these two inequalities, we obtain $\tilde{v}_2 \geq v_1$. Now suppose that $\tilde{v}_2 \geq v_1$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 2 obtains the object if and only if $b_2 = b_1 = v_1$ and $b_3 \leq v_3$.

Let us assume that ties are broken in favor of bidder 3. Suppose that there is a Nash equilibrium (b_1, b_2, b_3) where bidder 3 obtains the object. Then $b_3 = b_j$ for some $j \neq 3$. Suppose that $b_3 = b_2$. In order to prevent a deviation by bidder 3, we need to have $b_3 \leq v_3$. Since $v_1 > v_3$, bidder 1 can gain by deviating to $b_1 \in (b_3, v_1)$, a contradiction. Hence, it must be that $b_3 = b_1$. In order to prevent deviations by bidders 1 and 3, we need to have $b_3 \geq v_1$ and $v_3 - b_3 \geq e$, respectively. Combining these two inequalities, we obtain $\tilde{v}_3 \geq v_1$. Now suppose that $\tilde{v}_3 \geq v_1$. Then a bid profile (b_1, b_2, b_3) is an effectively undominated Nash equilibrium where bidder 3 obtains the object if and only if $b_3 = b_1 = v_1$ and $b_2 \leq v_2$. Since $b_3 = v_1 > v_2$, bidder 2 has no incentive to deviate.

(2) Consider a second-price auction. Let us assume that ties are broken in favor of bidder 2. At any effectively undominated Nash equilibrium where bidder 2 obtains the object and competes with bidder 1, bidder 2 chooses $b_2 = \tilde{v}_2$ and bidder 3 chooses $b_3 = v_3$. Since $v_1 > v_3$, there is an effectively undominated Nash equilibrium where bidder 2 obtains the object and competes with bidder 1 if and only if $\tilde{v}_2 \geq v_1$. If such an equilibrium exists, it is given by $(b_1, b_2, b_3) = (v_1, \tilde{v}_2, v_3)$, and bidder 2 pays the second highest bid v_1 at the equilibrium. At any effectively undominated Nash equilibrium where bidder 2 obtains the object and does not compete with bidder 1, bidders 2 and 3 choose $b_j = v_j$ for all $j \neq 1$. Since $v_1 > v_2 > v_3$, there is no such equilibrium.

Since $v_1 > v_2$, part (iii) for a second-price auction can be proven as in the proof of part (ii) above.

B Experimental Results on Incomplete Information

As explained in Section 3, we have implemented an incomplete information setting in our experiments, and we analyze our data from the incomplete information rounds in this appendix.

First, we check whether the incidence of overbids diminishes and that of efficient allocations increases as the rounds progress, in order to test whether bids and allocations converge to those at efficient equilibrium over time. Table 9 shows that there are no economically and statistically significant learning effects in these dimensions.

In Table 10, we compare the two auction formats in terms of the incidence of overbids, revenue, and the incidence of efficient allocations for the two considered parameter sets, one with a positive externality and the other with a negative externality. Columns (1) and (2) show that bidders overbid more frequently in second-price auctions than in first-price auctions. Columns (3) and (4) reveal that revenue is higher in second-price auctions than in first-price auctions. Columns (5) and (6) show that efficient allocations arise more often in first-price auctions than in second-price auctions, especially for the parameter set with a positive externality. Although these results are qualitatively similar to those on the complete information rounds, there are a few noticeable differences. First, revenue is higher in second-price auctions not only under the negative externality but also under the positive externality, and the magnitudes of the coefficients are quite similar between the two cases. Second, second-price auctions result in efficiency loss compared to first-price auctions under the positive externality, but not under the negative externality. Although we cannot generalize these differences due to the limited parameter sets used in the incomplete information rounds,[13] it seems that private information strengthens overbidding tendencies in second-price auctions especially under the positive externality, thereby leading to higher revenue and efficiency loss in second-price auctions under the positive externality.

Lastly, we summarize our results on the incomplete information rounds as follows.

[13] There was only a single set of parameters for each sign of externalities in the incomplete information rounds, whereas several parameter sets were used for each sign in the complete information rounds.

Table 9: Estimation Results for Learning Effects in the Incomplete Information Rounds

		Positive Externalities		Negative Externalities	
		FPA	SPA	FPA	SPA
Outcome Var.	Explanatory Var.	(1)	(2)	(3)	(4)
Overbids	Rounds	0.002	0.015	0.033	0.018
		(0.042)	(0.030)	(0.029)	(0.035)
	Observations	210	270	210	210
	Log-pseudo likl.	-133.0	-184.6	-97.3	-143.8
Efficiency	Rounds	0.068	-0.079	-0.037	-0.008
		(0.054)	(0.052)	(0.056)	(0.054)
	Observations	70	90	70	70
	Log-pseudo likl.	-47.2	-52.8	-44.8	-42.8

Note: Standard errors are clustered at the subject level in Overbids. Robust standard errors are in parentheses in Efficiency. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

Table 10: Estimation Results for Overbids, Revenue, and Efficiency in the Incomplete Information Rounds

Variables	Overbids		Revenue		Efficiency	
	Positive	Negative	Positive	Negative	Positive	Negative
	(1)	(2)	(3)	(4)	(5)	(6)
SPA	0.612***	1.086***	13.31***	14.87***	-0.700***	-0.120
	(0.218)	(0.289)	(2.336)	(3.091)	(0.206)	(0.221)
Observations	480	420	160	140	160	140
R-squared	-	-	0.151	0.144	-	-
Log-pseudo likl.	-317.7	-241.7	-	-	-102.2	-87.8

Note: Standard errors are clustered at the subject level in Overbids. Robust standard errors are in parentheses in Revenue and Efficiency. The notation *** indicates significance at 1% level, ** at 5% level and * at 10% level.

Result 5. *With incomplete information, there are no learning effects, and there are more over-bidding, higher revenue, and less efficient allocations in second-price auctions than in first-price auctions.*

C Experimental Instructions

Thank you for participating in the experiment. Please read the following instructions carefully.

Your decisions will be anonymously collected and used only for research. No one will know what your decisions are in the experiment.

You will obtain a gift certificate worth KRW 5,000 as a show-up fee. In addition to this show-up fee, you can earn an additional gift certificate whose value depends on your decisions as well as your group members' in the experiment.

You will participate in an auction for an item. Here is the rule:

- You are randomly grouped with others in this room to form a group of three. (Members do not know each other's identity.) The three members in a group participate in the auction. In each group, one member is called Red and the other two are called Blue.
- 170 coins are in your virtual account. You choose how many coins to bid out of 170 coins you have. The member with the highest bid wins. If there is more than one bidder who submits the highest bid, the winner is randomly chosen with equal chances.
- **[For the FPA treatment]** If you win, V coins are added to your account. (V can be different across members.) You pay your bid. In your account: 170 + V - [your bid].
- **[For the SPA treatment]** If you win, V coins are added to your account. (V can be different across members.) You pay the second highest bid. If there is more than one bidder who submits the highest bid, the second highest bid is equal to the highest bid. In your account: 170 + V - [the second highest bid].
- If you lose:

- If you are Blue and the winner is Red, E coins are added to your account. (E is the same for both Blue members.) If $E < 0$, this means that coins are subtracted from your account. In your account: $170 + E$.
- If you are Blue and the winner is another Blue, no change is made to your account. In your account: 170.
- If you are Red, no change is made to your account. In your account: 170.

You will play this auction for 25 rounds.

- The first 5 rounds are for practice. In each round, you are randomly re-grouped with others. These 5 practice rounds are not considered for payments. You have two minutes to make your decision in each round. (If you do not make your decision within two minutes, 0 will be entered as your decision.)
- The next 20 rounds are considered for payments. You have one minute to make your decision in each round.
    - For the first 10 rounds, you are randomly re-grouped with others in each round. Everyone knows the value of E and all the members' values of V.
    - For the second 10 rounds, you are randomly grouped with others in the first round and then the group stays the same for the 10 rounds. Everyone knows the value of E and his/her own value of V but not the others' values of V. That is, you do not know how many coins your group member obtains when he/she wins the auction. The value of V, which is fixed throughout the 10 rounds, is an integer between 30 and 100, and it can be different across your group members.

After the 25 rounds of auctions, the experiment ends. From the 20 rounds considered for payments, one round will be chosen randomly, and the total amount of your coins in that round will be converted to KRW 95 per coin and given to you as a gift certificate (in addition to your show-up fee).

Please do not talk with others nor use your phones. Please take your time when making your decisions in the experiment; you do not have to hurry.

If you have any question, please raise your hand. Please wait for further instructions.